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Industrial Machine Analysis: Economic Quality Control

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By

Vandyke Kotoroka-Yiadom

An Abstract of a Thesis
In
Applied Economics

Submitted in Partial Fulfillment
of the Requirements
for the degree of

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State University of New York
College at Buffalo
Department of Economics and Finance
ABSTRACT OF THESIS

Industrial Machine Analysis: Economic Quality Control

The objective of the study is to analyze the optimality of industrial machines over time in order to explore causes of variation in the machines’ performance. The accurate estimation of machine efficiency is very important in capital-intensive industry. This study selected various productions units, labor hours per pound, product mix and shifts, to analyze the variations in machine production.

This paper outlines methodology using statistical quality control methods, cluster analysis and econometrics to process and analyze the data. It then presents performance results of industrial machines and makes recommendations for future improvement of these machines. The firm that was analyzed is ideally positioned with about 6,300 employees at production sites in Europe, the USA and China. It is one of the foremost integrated groups with a leading position in recycling.

The data in this study are from a large North American manufacturing firm. These data are proprietary. The name and location of the firm, along with the exact subsector, are deliberately kept vague to protect the firm’s property and patent right.

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I dedicate this work to the memory of my grandparents, John Kofi Eduafo and Mary Conduah, from whom I learned the spirit of being patient and gentle, regardless of the circumstance and to my Dad, Kwame Kotoroka-Yiadom, who sponsored my residence in United States and ensured that I attended church every Sunday to worship God. Julia Eduafo, Mabel Adomako Ansah and Mrs. Blackburn have all been inspirational and supportive in my upbringing. Preeti Jaiswal and Martin Kotoroka-Yiadom were there when I needed the motivation to complete this thesis.

Subsequently, I would like to express the deepest appreciation to my committee chair, Dr. Frederick G. Floss, as well as Mrs. Liza Mohr, who was instrumental in securing critical data. They believed in me from the beginning of the thesis concept till completion. They were supportive educationally and professionally. Without them this thesis would not be possible.

I would like to thank my committee members, Dr. Theodore F. Byrley and John L. Wilcox. Dr. Theodore F. Byrley always reminded me to write a simple and precise thesis that will not keep me in school forever.
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i. **Introduction**

In recent years, more and more industries and organizations have become interested in “statistical quality control” (SQC) or “economic quality control” (EQC) as defined by Walter Shewhart.¹ Often the word “statistical is omitted from the expression by many people who think of statistical quality control as being applied only to the control of the quality of manufactured products. In this paper the phrase economic quality control will be used. The term EQC was coined by Walter Shewhart. Most prior research done in this area used statistical methods to achieve stable control in the quality of manufactured products; however, this research focused on achieving statistical stable control in the industrial machines used to produce these products. Management consultants and statisticians, such as Edwards Deming and Walter Shewhart, are promoting philosophies that contain strong statistical components and are being heard by top U.S. executives.

In order to survive and be able to provide customers with quality output, manufacturing organizations are required to ensure that their industrial machines are continuously monitored and output quality is improved. Quality of industrial machines may mean different things to different people. Quality deals mostly with people’s expectations and perceptions on how such expectations are being satisfied. Quality of industrial machine is a universal value and has become a global issue. Edwards Deming once said, “Every empirical scientific statement is a prediction, because, no matter how many times it has been confirmed in the past, it is always subject to future confirmation by experiment.”² Deming is regarded

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by the Japanese as the chief architect of their industrial success. According to Deming “all processes are vulnerable to loss of quality through variation: if levels of variation are managed, they can be decreased and raise quality.” This system of thinking is essential to the economic quality control of manufactured products, and it is principally in this field of work that Shewhart’s ideas have been validated and given extensive application.³

**Definition of Quality**

In establishing the state of quality control, it is imperative to proceed by explaining quality. Philip B. Crosby expresses quality as conformance to requirement (1980); Joseph M. Juran (1988) defines it as fitness for use; while Edwards Deming (1993) defines it as a predictable degree of uniformity and dependability at low cost and suited to the market. Crosby and Juran’s definitions focused on designing products and services that met customers’ needs and expectations. Crosby and Juran assumed that it is possible to identify a customer’s absolute needs and then develop quality guidelines to satisfy such needs.

According to Christian Madu, although this may be possible for products, it is often difficult to measure quality or identify attributes in service. Deming’s definition seems to be appropriate, since it allow room for variation in quality while guaranteeing a predictable degree of uniformity and dependability. Deming’s definition of quality focuses on achieving a stable process. If variations observed in a product are within the normally established limits, then the process is stable and can be considered to be predictable. This concept is in accordance with the definition of statistical quality control in the book *Statistical Method*

³ *Ibid*
from the View of Quality Control, authored by both Deming and Shewhart. Shewhart asserts that quality sought, to meet the needs of the consumer, and must be stated in terms of specified quality-characteristics that can be measured. It is necessary to predict what quality characteristics of products will produce satisfaction in use. Quality, however, to the consumer, is not a set of specifications. The quality of any product is interaction between the product, the user, his expectations and the service that he can get in case the product fails or requires maintenance. The needs of the consumer are in continual change. So are materials, methods of manufacture, and the manufacture products. Quality of a product does not necessarily mean high quality. It means continual improvement of the process, so that the consumer may depend on the uniformity of a product and purchase at low cost.

There is no doubt why Deming is regarded as the father of Total Quality Management. His earlier definition of quality focused on its statistical component. For the purpose of this research, we will focus on Deming and Shewhart’s later work that viewed quality from both statistical and economic perspectives.

Economic Quality Control and State of Statistical Quality Control

We begin by defining control by Shewhart: a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. The terms economic quality control and statistical quality control are used interchangeable by Shewhart. Shewhart (1980) defines economic quality control as the use of scientific method extended to take into account

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5 Ibid
modern statistical concepts to set up limits within which the results of routine efforts must lie if they are to be economical. Deviations in the results of a routine process outside such limits indicate that the routine has broken down and will no longer be economical until the cause of trouble is removed. His book *Economic Control of Quality of Manufacturing Product* investigates the scientific basis for attaining economical control of quality of manufacturing product through the establishment of control limits to indicate at every stage in the production process, from raw materials to finish products, when the quality of product is varying more than is economically desirable. Statistical methods of control have been developed by industry for the purpose of attaining economic control of quality of product in mass production.

Similarly, the state of statistical quality control is attained by statistical method, the establishment of tolerance limits, the presentation of data and the specification of accuracy and precision. Note that statistical control method is not simply the use of statistical technique, but the use of statistical technique that constitutes a means of attaining the end characteristics of the state of statistical control. Statistical methods of research have been highly developed in the field of agriculture. The distinction between the economic control of quality and statistical quality control is much clearer in its application.

The application of statistical methods in mass production makes possible the most efficient use of raw materials and manufacturing processes, effects economies in production, and makes possible the highest economic standards of quality for the manufactured product.
The economic control of quality of manufactured goods is the simplest type of scientific control.  

**History of Economic Quality Control or Statistical Quality Control**

The practice of economic quality control in the United States has rapidly grown since its inception and publication by Shewhart 70 years ago. This paper will trace its development and the future of industrial statistics.

Early in 1924, the awareness among Western Electric engineers and executives that the control of quality of manufactured products needed special investigation. This led to the formation of an Inspection Engineering Department, now the Quality Assurance Department of the Bell Telephone Laboratories. One of the first appointees to this group, then under the leadership of R. L. Jones, was Walter A. Shewhart, who was assigned to examine and interpret the inspection data covering production for the quarter year just completed. It was readily apparent to Shewhart that little useful inference could be drawn from the data, except that something serious should be done. Shewhart presented his first memorandum on the control chart and began his systematic development of the theory and practice of statistical quality control, which was well rounded out by 1929. The first published description of the control chart appeared in *The Bell System Technical Journal*.

Karl Daevs’ “Grosszahlforschung” dealt with frequency distributions of observations made in steel production, but neither he nor any other statistician provided a means for determining whether one had a distribution. Considerable reference was made in the literature

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6 Ibid


8 Ibid
to the assumption of the existence of distributions and of randomness, but no reference was made as to how to obtain a distribution or state of randomness. In 1924 Shewhart showed (operationally) how to determine that the source of one's observations behaved as though a distribution existed and how to bring about this state. Shewhart was not aiming at the development of statistical theory but at a “practical” goal, namely getting some information out of inspection data so as to obtain uniform manufactured products economically.\(^9\)

In accomplishing this specific objective, he formulated the general concepts of statistical quality control, which represented a great step forward in the use of statistical method. It was a major advance in experimental method marked by the fundamental fact that the attainment of control required the setting of goals. The methods of statistical quality control have experimental meaning relative to randomness in that the test for a state of control relative to a given goal is as far as one can go in testing randomness.

Shewhart wrote down the essential developments of his ideas and published them.\(^10\) The soundness of the methods of statistical quality control was validated by “an exacting testing program” carried on by the staffs of both the Bell Telephone Laboratories and the Western Electric Company over a period of years. SQC was made a part of the regular procedures of the production divisions; it was with reasonable assurance that it would work effectively.\(^11\)

\(^9\) Ibid
\(^10\) Ibid
\(^11\) Ibid
Sequential Account of Economic Quality Control

Juran expounded the concept of statistical quality control in his memoirs of the mid-1920s. Juran states:

“...as a young engineer at Western Electric’s Hawthorne Works, I was drawn into a Bell Telephone Laboratories initiative to make use of the science of statistics for solving various problems facing Hawthorne’s Inspection Branch. The end results of that initiative came to be known as statistical quality control or SQC.”

It is clear from the above statement that the concept of statistical quality control had its inception in the Bell Laboratories in the mid-1920s. As the Bell Telephone Company was rapidly expanding in the mid-1920s, it was faced with various quality problems resulting from its large production of telephone equipment. According to Juran, the end result of this initiative came to be known as SQC. Table 1 provides a summary of the milestones in the history of statistical quality control.

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Table 1: Selected Historic Milestones in the Quality Movement

<table>
<thead>
<tr>
<th>Year</th>
<th>Milestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>1924</td>
<td>Walter Shewhart developed the control chart.</td>
</tr>
<tr>
<td>1931</td>
<td>Walter A. Shewhart of Bell Laboratories introduced statistical quality control in his book <em>Economic Control of Quality of Manufactured Products</em>.</td>
</tr>
<tr>
<td>1940</td>
<td>W. Edwards Deming assisted the U.S. Bureau of the Census in applying statistical sampling techniques.</td>
</tr>
<tr>
<td>1941</td>
<td>W. Edwards Deming joined the U.S. War Department to teach quality-control techniques.</td>
</tr>
<tr>
<td>1951</td>
<td>Joseph M. Juran published the <em>Quality Control Handbook</em>.</td>
</tr>
<tr>
<td>1954</td>
<td>Joseph M Juran addressed the Union of Japanese Scientists and Engineers (JUSE).</td>
</tr>
<tr>
<td>1968</td>
<td>Kaoru Ishikawa outlined the elements of Total Quality Control (TQC).</td>
</tr>
<tr>
<td>1970</td>
<td>Philip Crosby introduces the concept of zero defects.</td>
</tr>
<tr>
<td>1979</td>
<td>Philip Crosby publishes <em>Quality is Free</em>.</td>
</tr>
<tr>
<td>1980</td>
<td>Western industry began to import the concept of TQC under the name Total Quality Management (TQM).</td>
</tr>
<tr>
<td>1980s</td>
<td>American electric giant, Motorola, pioneered the concept of Six Sigma.</td>
</tr>
<tr>
<td>1986</td>
<td>W. Edwards Deming published <em>Out of Crisis</em>.</td>
</tr>
<tr>
<td>1987</td>
<td>U.A. Congress created the Malcolm Baldrige National Quality Award.</td>
</tr>
<tr>
<td>1988</td>
<td>Secretary of Defense Frank Carlucci directed the U.S. Department of Defense to adopt total quality.</td>
</tr>
<tr>
<td>1993</td>
<td>The total-quality approach began to be widely taught in the U.S. universities</td>
</tr>
</tbody>
</table>
Control Chart (1924)

The control chart was invented in the Bell Labs by Walter Shewhart on May 24, 1924. Graphically represented by plotting process parameter against time, a control chart is intended to monitor process stability and variability. The graph includes a tolerance, an upper control limit and a lower control limit. A control chart is one of the most important SQC methods in quality control and improvement. It is a proactive statistical tool intended to monitor processes and signal when they go out of control.

Acceptance Sampling Plans (1925)

Developed in the Bell Telephone Company in 1925 by Harold Dodge, acceptance sampling plans are reactive SQC tools, in the sense that they are used to ensure the quality of the finished products. In 1940, the US Bureau of the Census used statistical sampling techniques with the assistance of Edwards Deming. Acceptance sampling plans gained usage during World War II, when the both British and US Military adopted the standardized product sampling inspection schemes and required their suppliers to meet these schemes. Meanwhile, in 1941, Edwards Deming joined the US Army to teach quality control techniques and it was during this time when the Bell Lab developed for the US Army the Sampling Procedures and Tables for Inspection by Attributes. These Sampling Procedures and Tables, published for general use in 1944, are called the US Military Standard 105A (MIL-STD-105A).
The Cause-and-Effect Diagram / the Ishikawa Diagram (1943)

Also known as the fishbone diagram, the cause-and-effect diagram is a graphical statistical tool that illustrates the causes of variations in product quality. This diagram was introduced into the realm of quality control by Kaoru Ishikawa in 1943. Ishikawa identified four broad categories of causes of variations including variations, in materials, processes, equipment, and measurement. The cause-and-effect diagram focuses on the improvement of material, equipments and processes in order to ensure quality control. It was only in the 1970s when this SQC tool gained expanded use in Japan.\(^{13}\)

**Statistical Thinking in Japan**

Statistical quality control was first heavily implemented in post-World-War-II, Japan in the late 1940s and the early 1950s. By the late 1940s, Japanese top management and engineers began to realize that quality was the key to gaining competitive advantage in the world marketplace.

The Union of Japanese Scientists and Engineers (JUSE) was organized by Ichiro Ishikawa, the father of Kaoru Ishikawa, in 1946 with a view to revitalize Japan’s economy and eliminate waste by improving quality. In 1949, JUSE hosted its first seminar on statistical quality control. In 1950, JUSE invited Edwards Deming to lecture the Japanese Scientists and Engineers on the use of SQC. Deming inspired them to employ SQC thinking to address manufacturing quality problems. The notes to Deming’s lecture are known as Elementary Principles of the Statistical Control of Quality. Deming refused the royalties

\(^{13}\) Hossain, Muhammad. "Development of Statistical Quality Control: Evolution or Revolution." University of North Texas, n.d.
offered to him by JUSE for his lectures. JUSE established the Deming Prize with the royalties and awards the prize to those who contribute to the field of statistical quality control. Later in 1954, JUSE invited Juran who emphasized the vital role of statistical methods in managing quality of the manufacturing sector. Juran’s ideas about statistical quality management began spreading in Japan and emerged as total quality control in the late 1960s.

**Deming Cycle (1950)**

Deming’s work on quality control was considerably influenced by the work of Shewhart. In 1950, Deming came up with what is now called the Deming Cycle of Quality Control. The Deming Cycle analyzes and measures business processes to identify the sources of deviation. The processes are then placed in a continuous feedback loop for managers to detect defects and improve the processes as required.

**The Juran’s Trilogy (1951)**

Joseph M. Juran developed the Juran Trilogy in 1951 as a quality control tool and published it in his book Quality Control Handbook. The Juran Trilogy encompasses three managerial functions – quality planning, quality control, and quality improvement – geared towards ensuring product or service quality. It emphasized the importance of system thinking and laid the foundation for transition from SQC to total quality control (TQC) in Japan.
Total Quality Control (TQC) / Total Quality Management (TQM) (1960s – 1980s)

Juran’s lectures on statistical quality control in 1954 and 1960 organized by JUSE focused on managing quality and applying quality as a company-wide business strategy. These lectures were in line with JUSE’s theory of continuous improvement and quality circles, and provided the underlying principles of total quality control. However, it was Kaoru Ishikawa who had outlined the elements of TQC in 1968. In the early 1980s, the US companies began importing both what Deming and Juran exported to Japan earlier and what Japanese quality philosophers developed by the 1970’s, such as TQC and Just-In-Time manufacturing. The concept Total Quality Control that originated in Japan came to be known as Total Quality Management in the western world. TQC and TQM have their roots in statistical quality control, and are, therefore, considered integral parts of SQC.

Zero Defects (1979) and Six Sigma (1980s)

The concept of Six Sigma was pioneered by Motorola, an American electronics giant, in the early 1980s. Six Sigma has its roots in the notion of zero defects. The term “zero defects” was coined by Philip Crosby in his 1979 book Quality is Free. The concept of zero defects revolves around doing things right the first time and reducing the defects to zero. Reducing the number of defects to zero may not occur in practice. However, the notion of zero defects does not, in absolute sense, mean that no errors will take place. Rather it emphasizes the fact that everything possible has been done with a view to eliminate the defects from occurring.

Six Sigma is defined by Roy as “the improvement or design/redesign of business processes to meet exactly customer requirements, to offer products, which are 100% compliant to the customer-related specifications, produced at minimum costs.” It is a
methodology that is well rooted in statistics. The objective of Six Sigma is to reduce process output variation so that this will result in no more than 3.4 defect parts per million (PPM) opportunities (Sixsigma.com).

Given the specification limits, the variation of the process around the mean value keeps decreasing as the process standard deviation value keeps increasing from zero. Six Sigma uses this notion of statistics to achieve near perfection. It uses six process standard deviations between the mean value of the process and the customer’s specification limit, resulting in no more than 3.4 defective parts per million (PPM) opportunities.

In a nutshell economic quality control, which was further developed through scientific and statistical methods, has a strong historical foothold. The modern statistical quality control philosophies such as total quality management, zero defects and Six Sigma, are striving to achieve organization-wide error-free quality products and services. Economic quality control is an integral part of the paradigm of quality.
ii. Literature Review

This paper adopts econometrics and statistical control methods to attain the state of economic quality control for industrial machines in the firm analyzed. Economic control of quality is the use of the scientific method that set up limits within which the results of routine efforts must lie if they are to be economical. Our focus is to compare similar literature in this area of research and make suggestions that contribute to the quality control movement. Engineers and statisticians apply various quality control techniques to improve the quality of process by reducing its variability. It is a popular misconception that industrial machines will produce identical output. Unfortunately, real life considerations interfere with this theoretical ideal. The output of machines may vary due to maintenance time, setups, different shifts, operators etc., and such random variables may affect productivity. In this study the variation in output per unit of each machine will be measured and compared to industry, department and company standards.

Shewhart’s Control Chart

People who are fascinated by the study of economic quality control will appreciate the work of Dr. Shewhart. To Shewhart, quality control meant every activity and every technique that can contribute to better living, in a material sense, through economy in manufacturing. One of the purposes of this paper is to emphasize the need for continual search for better understanding about industrial machines, and how they behave in manufacture. Economic manufacturing requires achievement of statistical measure. It requires improvement of process in every feasible way. The cost and inadequacy of
inspection are well known. Solving this problem begins with the use of a statistical tool called the control chart.

Shewhart has completed extensive research on economic control of quality using data from the Bell Labs. Shewhart’s studies focused on two major areas: indicating the presence of assignable causes of variation in each of the quality characteristic and indicating the seriousness of the trouble and the steps to eliminate it. In order to detect the presence of assignable causes Shewhart suggested establishing the control chart, including allowable limits on the variability of sample. If more than one statistic is used, then the limit on all the statistics should be chosen so that the probability of looking for trouble when any of the chosen statistics falls outside its own limits is identified, even when no trouble exits, Shewhart encouraged researcher to look for trouble, nevertheless. On the other hand, the smaller the probability, the more often in the long run, researcher may expect to catch trouble if it exists.

The principal function of the control chart is to detect the presence of assignable causes. An assignable cause of variation, as this term is used in quality control work, is one that can be found by experiment without costing more than it is worth to find it. Defining an assignable cause today might not be one tomorrow, because of a change in the economic factors of cost and value of finding the cause.

Shewhart’s control chart idea, in his book Statistical Method from the Viewpoint of Quality Control, took 204 observations of insulation resistance, which was used to illustrate some of the characteristics of a control chart as a tool for detecting the presence of assignable

\[14\text{ Ibid}\]
cause. Shewhart grouped this observation into subgroups of four taken in order which the observation were made, and applying the control chart concept to 51 subgroup averages. Figure 1 is Shewhart’s control chart.

Exhibit 1

![Control Chart](image)

Where UCL is the upper control limit, CL is the control limit and LCL is the lower control limit. Here, he indicates the presence of assignable causes of variability, which further research revealed and removed. Shewhart emphasizes the importance of order considering the insulation material observation. Consider if we had not known the order in which the 204 pieces of insulation material were made. For example, suppose that these pieces of insulation had been thoroughly mixed together in a box before the measurements of resistance had been made, as is a very common practice.

The 204 measurements of resistance on the 204 pieces of material after they had been thoroughly mixed would have been the same, but we do not know anything about the order in which the pieces were made. Of the 204 different orders that might be obtained by such random operation, the order of manufacturing becomes the basis of the control chart.

Shewhart asserts that instead of mixing the pieces of insulation material in a box and measuring one piece at a time upon drawing it, we may write the 204 original measurements
on as many physically similar chips, mix the chips in a bowl, draw them one at a time without replacement. Suppose we apply the same ideas of sub-grouping to one sequence of 204 numbers obtained in this way. The results would be the lower half of figure 1. There would be no indication of assignable causes. If in this case, the original order had not been given and we had taken instead the order actually given by the random operation of drawing the 204 number one at a time from a bowl, the application of sub-grouping would have failed to detect the presence of assignable causes. It is shown theoretically that if we apply sub-grouping in the same way to all of the 204 observation, in a possible different sequence, most of them would give no indication of the presence of assignable causes in the sense of showing averages. We must note that the original sequence is one of the 204 possible sequences generated by such a random operation. Hence, the failure to meet the criteria does not serve to pick any one of the 204 observation sequence drawn from a bowl as being nonrandom because in fact, they all are obtained by means of a random operation.\textsuperscript{15} Why then should we place faith in a sub-group indicator of assignable causes?

Extensive research has shown that one almost never finds in practical work an observed sequence, even when obtained under presumably the same essential conditions that will satisfy sub-grouping, and assignable causes are looked for when an observed statistic goes outside the limits. If removing assignable causes is continued, we gradually approach a condition where observed statistics only seldom goes outside of its limit, and if one looks for assignable causes in these rare instance, such causes are not usually found. Shewhart emphasizes that the importance of sub-grouping to detect the presence of assignable causes should always focus on breaking up the original sequence into sub-groups of comparatively

\textsuperscript{15} Ibid
small size. If this is not done, the presence of assignable causes will very often be overlooked.

Control charting can signal the need for process intervention and can keep one from ill-advised and detrimental over-adjustment of a process that is behaving in a stable fashion. But, in doing so, what is achieved is simply reducing variation to the minimum possible for a given system configuration (in terms of equipment, methods of operation, methods of measurement, etc.).

**Composite Exponentially Weighted Moving Averages (CEWMA)**

Zhang, Bebbington, Govindaraju & Lai (2007) have a different perspective about the control chart. They propose a new charting procedure, combining two or more of what they term Exponentially Weighted Moving Averages (EWMAs), which is called the Composite EWMA (CEWMA) control chart. In the case of two EWMAs, the CEWMA chart corresponds to the usual combined Shewhart-EWMA chart: They believe that the CEWMA is an improvement in the application and interpretation. When three EWMAs are combined, the composite chart improves on the average run length performance of the combined Shewhart-EWMA charts. They claim that EWMA chart is quick to detect small persistent process shifts but does not react to large shifts as quickly as the Shewhart chart.

Zhang, Bebbington, Govindaraju & Lai, C. (2007) assert that the combined Shewhart-EWMA control chart is rather complicated as both the Shewhart and EWMA control statistics need to be plotted along with their control limits. To enhance the sensitivity of the EWMA chart to large shifts, Lucas and Saccucci (1990)\(^\text{16}\) suggested a combined use of the

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Shewhart-EWMA control charts. In this procedure, both EWMA and Shewhart control statistics are plotted; if either statistic exceeds its own control limits an out-of-control signal is given. Woodall and Maragah (1990)\(^\text{17}\) further emphasized that a Shewhart chart should always be used in conjunction with the EWMA chart, and Klein (1996)\(^\text{18}\) showed that the combined (or composite’ the term used by the author) Shewhart- EWMA control charts have a better average run length (ARL) performance than the Shewhart chart supplemented with run rules.

Zhang, Bebbington, Govindaraju & Lai complement Shewhart’s chart as a special case of the EWMA chart (with smoothing constant \(\lambda = 1\)). They proposed a new chart, in which two or three EWMAs (one of which is the EWMA with \(\lambda = 1\)) are combined using a different approach. They call this new chart a composite EWMA (CEWMA) control chart, and showed that the CEWMA chart is efficient, easy to apply and interpretable. They assumed that the output of a process of interest is characterized by a random variable \(X\), which has a normal distribution with mean \(\mu_0 + \delta \sigma_0\) and variance \(\sigma_0^2\). Both \(\mu_0\) and \(\sigma_0\) are known but \(\delta\) is unknown. If \(\delta = 0\), then the process is in control; otherwise, the process has changed or shifted. They further assumed that samples obtained at time \(t=1, 2, \ldots\) are mutually independent.

For time \(t = (t \geq 1)\), when the current observation is \(x_t\) observation can also be a sample mean from a rational subgroup, and in such a case \(\sigma_0\) represents the standard deviation of the sample mean. They define control statistic, \(Q_t^{(1)}\) as:

\[ Q_t^{(1)} = \lambda_1 y_t + (1 - \lambda_1)Q_{t-1}^{(1)} \]

Theoretically, \( \lambda_0 \) can be any value in the interval \((0, 1)\); however, they choose to \( \lambda_0 \) equal to 1, because by doing so the Shewhart charting mechanism, which is simple, will make their chart quickly detect large sporadic process changes. Zhang, Bebbington, Govindaraju and Lai argue that out-of-control signals must result from one of the following cases:

Plot a RED point distance (e.g., \( \sigma_0 \) units) above the UCL or below the LCL, and suspect that a large process shift (or shock, \(|\delta| \geq 2\)) has occurred.

Plot a BLACK point at \((t, Q_t^{(1)})\), and suspect that a relatively small process shift has occurred. If neither of the two cases above occurs i.e., if \( Q_t^{(1)} \in [LCL, UCL] \), then we plot a GREEN point at \((t, Q_t^{(1)})\), and regard the process as being in control.

The analysis of the CEWMA chart is more complex than that of the combined Shewhart- EWMA charts. The EWMA control procedure can be made sensitive to a small or gradual drift in the process, whereas the Shewhart control procedure can also react when the last data point is outside the control limits. They argue that the CEWMA chart is more sensitive to large process shifts (\(|\delta| \geq 2\)) than the EWMA chart, and slightly less sensitive to process shifts with \(|\delta| < 1\) than the EWMA chart. They conclude that, overall, the CEWMA represents a very good ARL performance.
Double Generally Weighted Moving Averages (DGWMA)

Shey-Huei and Hsieh (2008)\(^\text{19}\) extend the generally weighted moving average (GWMA) control chart by imitating the double exponentially weighted moving average (DEWMA) technique. Their proposed chart is called the double generally weighted moving average (DGWMA) control chart. They employ simulation to evaluate the average run length characteristics of the GWMA, DEWMA and DGWMA control charts. They reveal that the DGWMA control chart with time-varying control limits is more sensitive than the GWMA and the DEWMA control charts for detecting medium shifts in the mean of a process when the shifts are between 0.5 and 1.5 standard deviations.

Research has shown that GWMA control chart is more sensitive in detecting small shifts in the process mean, and can spot small shifts in the initial process, due to its added adjustment parameter \(\alpha\). They also showed that the composite Shewhart–GWMA control charts with or without runs rules is more sensitive than the GWMA control chart in detecting small shifts. Extensive research was done on the EWMA control chart to a double exponentially weighted moving average (DEWMA) control chart and also revealed that DEWMA has better performance than EWMA in small mean shifts. Shey-Huei and Hsieh (2008) argued that their analytical result indicate that the DGWMA control chart is more sensitive than the GWMA and DEWMA control charts for detecting medium mean shifts between 0.5 and 1.5 standard deviations.\(^\text{20}\)

The DGWMA control chart has up to four parameters: The calculation is complicated and even inconvenient, but, specifying values for some parameters can decrease the number


\(^{20}\) Ibid
of DGWMA parameters from four to two, or even one, thus reducing the calculation complexity, and significantly improving practicability. Shey-Huei and Hsieh (2008) emphasized that in order to understand the performance of the DGWMA control chart thoroughly, the ARL values for the DGWMA control limits for specified values of the limit are provided. The DGWMA \((q, \alpha)\) control chart is designed to choose the pair \((q, \alpha)\) appropriately under these considerations. The ARL is the number of points that, on average, will be plotted on a control chart before an out of control condition is indicated. The ARL of the DGWMA charts with \(\text{ARL}_0 \approx 500\) when the design parameter \(q \in \{0.7, 0.8, 0.9\}\) and the adjustment parameter \(\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9, 1.0\}\). They claim that the DGWMA charts with \(\alpha \approx 0.5\) offer the better performance in detecting shifts. In the DGWMA chart with larger \(q\) and \(\alpha \approx 0.5\), the \(\text{ARL}_1\) becomes much smaller, but the standard deviation run length \((\text{SDRL}_0)\) is enlarged substantially.\(^{21}\)

Their investigation proposed that the DGWMA control chart for monitoring the process mean and deriving the ARL of the DGWMA control chart through simulation, indicates that the DGWMA control chart is superior to the GWMA and DEWMA control charts in detecting medium shifts of the process mean. When the shifts are less than 0.5\(\sigma\), the DGWMA chart is more sensitive than the DEWMA chart, but slightly insensitive compared with the GWMA chart. Subsequently, when the shifts are between 0.5\(\sigma\) and 1.5\(\sigma\), the DGWMA chart is superior to the GWMA and DEWMA charts. Lastly, when mean shifts are larger than 1.5\(\sigma\), the DEWMA chart is the best choice.

\(^{21}\)Ibid
Account of Basic tools of Statistical Process Control (SPC)

Box and Narasimhan (2010) provide an elementary account of the basic tools and applications of statistical process control (SPC). Their approach follows ideas from Box and Jenkins (1962), Box et al. (2008). They also incorporate some elementary concepts from engineering process control (EPC), in particular the ideas of nonstationarity and feedback. Because of the complementary roles played by SPC and EPC, their resulting technique has been called synergistic control. Box and Narasimhan begin their article by focusing on random and sequence sample, which is emphasized by Shewhart.

Box and Narasimhan assert that the key idea is that process data are some kind of sample. A random sample of size n is such that the data are independently distributed, and all possible samples of size n from the assumed generating distribution have equal probability of occurrence. However, data from an industrial process with observations made at, for example, every hour are unlikely to be random. They are in a particular order, and they form sequential samples. A sequential sample of size n is such that only sequences of length n from an assumed generating time series are equally probable. Data from such a sequence are almost certainly dependent and though the variance of all differences taken m steps apart is constant that constant is, however, different for different values of m. Thus, as was pointed out by Shewhart (1931), their order of occurrence is vital to understanding such data. Notice, however, that if we mistakenly assume that such a data sequence is a random sample, then the variance of the difference between the observations, however far apart, will be constant.  

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23 Ibid
Box and Narasimhan emphasized that, to obtain control limits for quality control charts, it was originally supposed that process data in a “state of control,” although observed in a particular order, could be treated as a random sample from a distribution with fixed mean and standard deviations. More specifically, they were supposed normally identically and independently distributed (NIID). They claimed that Alwan and Roberts (1988) pointed out that for a process data, a formulation that allowed for data dependence, such as the auto-regressive model, was more realistic.

Auto-regressive models provide a way of relating data to a generating white noise series. A different way of doing this uses a moving average (MA) model— for example, a first-order moving average. More elaborate moving average models employ additional terms—for example, a second-order moving average. If the order of autoregressive process is known, in practice, you need to test for the presence of autocorrelation. The Durbin-Watson test is a widely used method of testing for autocorrelation. This statistic can be used to test for first-order autocorrelation.

Box and Narasimhan suggest what a good model is. A good model approximates relevant characteristics of the phenomenon under study and is parsimonious and robust. Parsimony requires that you keep to a minimum number of unknown parameters in the model and so minimize transmitted variation from their estimates. The “improved” fit gained by increasing the number of parameters is often illusionary. The robustness of a procedure to the violation of a particular assumption is such that, even though the assumption can be a long way from reality, the derived procedure is not much affected. Thus, for example, the
investigation of Alwan and Roberts demonstrated that the positioning of Shewhart control limits is not robust to the assumption of data independence.\textsuperscript{24}

Box and Narasimhan claim that stationary process models assume that, if we left a process alone, it would continue to vary about some fixed mean with a fixed standard deviation. They provide reasons for uncontrolled process data to be nonstationary. The second law of thermodynamics states that the entropy (disorder) of any system can only increase. Thus, suppose it was possible to initially arrange that at some particular time, a process was operating perfectly; subsequently, in the absence of adjustment, it would inevitably deviate from this ideal.

A purpose of quality control is to induce a stable process, but stationarity assumes that the process is already stable and that the data vary about a fixed mean. A model should reflect what we fear, not only what we hope is true.\textsuperscript{25}

Even if a system were assumed to be stationary, the mean would need to be estimated from previous data. However, a large number of previous observations would be necessary to achieve ‘‘sufficient precision.’’ Also, this supposition places a great strain on the assumption of stationarity because it supposes that the average over some previous set of data is still relevant at the present time. In any case, the objective should not be to bring the process close to the unknown mean but to the target value, which is known exactly. If for some particular data sequence, a stationary model and a nonstationary model with the same number of parameters produced an almost equally good fit, it would almost always be better to use the nonstationary model to allow for later instability.

\textsuperscript{24} Ibid
\textsuperscript{25} Ibid
Box and Narasimhan conclude that EWMA is robust and simple. To get started you need only to know the target value $T$, to choose in consultation with management a value for $L$, and to make an educated guess for $\lambda$. The behavior of the control scheme is quite robust to the choice of $\lambda$ and operation of the process generates information from which $\lambda$ and $\sigma$ may be estimated. They further argue that one simple way to check the need for a more elaborate model is to use, as an approximation, the ‘‘extra sum of squares’’ principle developed for models in which the parameters appear linear.

Shewhart has contributed significantly to the development of economic quality control through the development of the control chart. He pioneered the idea of indicating the assignable cause and emphasized eliminating it. Shewhart’s control chart is often used with the EWMA to indicate variations in the manufacturing processes. Shewhart’s control chart is simple and not robust, unlike the EWMA. However, the combined model provides exceptional result of detecting assignable causes of variation. As a beginner, studying quality control, the Shewhart control chart provides detailed understanding of establishing stable control in the manufacturing processes. Shewhart highlighted the practice of continuously looking for trouble even if the current probability fell within the economic limits. Because the assignable cause today might not be one tomorrow if economic factors changed, he further emphasize the importance of sub-grouping as well as maintaining the original sequence of manufacturing data to detect the presence of assignable causes. Shewhart control chart only indicates a danger when a point falls outside the control limits. Shewhart’s charting mechanism is too simple and makes it difficult to quickly detect large sporadic process changes.
The DGWMA model propounded by Shey-Huei and Hsieh seems more complex than the Shewhart control chart. As a beginner, it will be difficult to comprehend with the statistical calculations involved in establishing the DGWMA control chart. Although it is complex and robust, it is more sensitive with time-varying control limits to detecting medium shifts in the mean of a process when the shifts are between 0.5 and 1.5 standard deviations.

Zhang, Bebbington, Govindaraju and Lai aimed at simplifying the Shewhart-EWMA control chart by introducing the CEWMA control chart. The CEWMA is significant in detecting the average run length (ARL) performance. The CEWMA has two or three EWMA s (one of which is the EWMA with $\lambda=1$) combined with a different approach. However, the average run lengths (ARLs) for comparing control chart performance follows a geometric distribution, which has high variability.

Box and Narasimhan discuss the elementary account of the basic tools and applications of statistical process control. They suggest a sampling concept that is similar to Shewhart’s view, thus the idea of maintaining the original sequence of manufacturing data in order to detect assignable causes of variation. According to their work, the EWMA is a good model. This is not surprising because they narrow their exploration to examining only the integrated moving averages (IMA), stationary and nonstationary models. They fail to consider the depth of Shewhart’s control chart, the CEWMA and the DGWMA control charts. They focus on selecting a good model based on its robustness and parsimonious.

Shewhart’s emphasis on random and sequence distribution as a key to establish economic quality control is highly debated. Zhang, Bebbington, Govindaraju & Lai (2007)
disagreed and stated that the sequence of manufacturing data (123…) is mutually independent. Although Box and Narasimhan disagree with Shewhart’s control chart based on robustness, they both argue that manufacturing data are dependent and is the basis for quality control chart.

Zhang, et al, also claim that EWMA is a good model because of its robust and simple. They suggest that Shewhart control chart is complicated because of the control limits it incorporates. However, their control chart procedures in detecting shift in performance is rather complicated. They distinguish this from Shewhart’s chart with mere colors.

Our control chart used one standard deviation, which is similar to the DGWMA control chart by Shey-Huei and Hsieh. They assert that, when the shifts are less than 0.5\(\sigma\), the DGWMA chart is more sensitive than the DEWMA chart, but a little insensitive compared with the GWMA chart. The DGWMA would have been simple if there were limited parameters and calculations to deal with. However, it helps to detect medium shifts in the mean of machine performance.

**SLT Department -Moving Average Approach**

The literature of control charts indicates a progression towards weighted moving average (WMA) theories. WMA helps to detect small to medium shifts from mean and average run length. This study applied the moving averages approach to machine/unit 001 of the SLT Department by dividing the data into four sub-groups. Unit 001 has 72,457 observations and is divided in sub-groups of about 18,114 observations. The assumption here is to predict whether the deviations within these sub-groups are constant. The following denotes the formula:
The mean and standard deviation of the sub-groups are as follows;

Where \( \hat{X} = Moving\ average \)

\( X_t \) is 1st mean = 1.9

\( \sigma_t \) is 1st deviation = 1.0

\( X_{t-2} \) is 2nd mean = 1.8

\( \sigma_{t-2} \) is 2nd deviation = 1.0

\( X_{t-3} \) is 1st mean = 1.7

\( \sigma_{t-3} \) is 1st deviation = 1.0

\( X_{t-4} \) is 1st mean = 1.7

\( \sigma_{t-4} \) is 1st deviation = 1.0

\[
\hat{X} = \frac{X_t - X_{t-2} - X_{t-3} - X_{t-4}}{\sigma_t + \sigma_{t-1} + \sigma_{t-2} + \sigma_{t-3}} \div 4
\]

\( \hat{X} = -0.206 \)

The moving average of unit 001 is -0.206, meaning it’s constant. Based on this result we are able to construct a linear mean \( \mu \) or tolerance for the SLT Department.
iii. Empirical Data

The data set contains 639,569 machine observations between 2011 and 2012 at the Northeast American firm. The data was obtained from the firm’s Enterprise Resource Planning (ERP) systems. The ERP systems record all the production details in the plant. Originally, twenty-seven machines were selected and later reduce to eighteen. Nine of machines were eliminated due to insufficient details of production data. Machine performance is measured by pounds per output and is categorized as sequence sampling because the sample is obtained through manufacturing processes. The observations obtained from the ERP systems are grouped into ten sub titles: individual operators, years of experience (YOE), pounds, machine/unit, department, shifts, start time, end time, gauge, and next unit/machine of operation. These data are categorized as nominal discrete, ordinal, ratio and Interval (e.g. types of machine, shifts, time, and pounds).

Methodology

Cluster analysis is a statistical technique that sorts observations into similar sets or groups. Shewhart emphasized that in order to detect assignable causes of variation, sub-grouping of observation is critical. Initially, the 639,569 observations were sub-grouped into individual machines and departments using a pivot table. The total number of machines and departments after organizing the data into clusters were eighteen and five respectively. The five departments were, for confidentially purposes, termed RER, RUL, SLT, CNT and STD and the eighteen machines were 001, 022, 023, 036, 027, 040, 044, 046, 047, 055, 068, 073, 074, 077, 078, 133, 143 and 147. It is important to note that in order to detect the presence of assignable causes of variation, you should emphasis always breaking up the original sequence
of data into sub-groups of comparatively smaller sizes. If this is not achieved, the presence of assignable causes will very often be overlooked.

Procedure

Statistical modeling is used to establish the $\mu$ mean and $\sigma$ standard deviation of the data grouped into the department and machine. To obtain a smaller sample sizes, we use the following formulas; Industrial specification

i. \[
Z\text{-test (}Z_i\text{)} = Z_i = \frac{X_i - \mu}{\sigma_{xi}}
\]

This compares the pounds produced by each machine that performed during 2011 to 2012 to the mean and standard deviation of the total industrial specification. The following steps are involved in obtaining the industrial specification Z-test:

- Group the output (pound) into actual days that the machines performed.
- Identify the mix adjusted pound per hour of each machine
- Multiply the adjusted pound per hour of each machine by 24 hours
- Calculate the mean, variance and standard deviation of the results attained in step 3.
- And then use this formula to compute the industrial specification Z-test, $Z_i = \frac{X_i - \mu}{\sigma_{xi}}$

Where

$Z_i =$ Z-test for the industrial specification

$X_i =$ Pounds per individual observation

$\mu =$ Mean of the industrial specification

$\sigma_{xi} =$ Standard deviation of the industrial specification
i. Company Z-test ($Z_C$) = $Z_C = \frac{X_i - \mu}{\sigma_{xi}}$

Where

$Z_C$ = Z-test for the company

$X_i$ = Pounds per individual observation

$\mu$ = Mean of the company

$\sigma_{xi}$ = Standard deviation of the company

Company Z-test compares the total pounds output of each machine to the mean and standard deviation of the total company actual pound.

ii. Machine Z-test ($Z_M$) = $Z_M = \frac{X_i - \mu}{\sigma_{xi}}$

Where;

$Z_M$ = Machine Z-test

$X_i$ = Pounds per individual observation

$\mu$ = Machine mean

$\sigma_{xi}$ = Machine standard deviation

This compares the total machine pound to the mean and standard deviation of the total machine output.

iii. Department Z-test ($Z_D$) = $Z_D = \frac{X_i - \mu}{\sigma_{xi}}$

Where;

$Z_D$ = Department Z-test

$X_i$ = Pounds per individual observation

$\mu$ = Department mean
$\sigma_{xI} =$ Department standard deviation.

The department $Z$-test compares the pounds per machine to the mean and standard deviation of the total department output.
Table 1 – An Example of Z-test Result

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<td>3.4979</td>
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Empirical data Machine 44-2011-2012
Table 1 represents results of 30 observations of the department Z-test ($Z_D$), company Z-test ($Z_C$), machine Z-test ($Z_M$), and industrial specification Z-test ($Z_I$).

The company Z-test ($Z_C$) is derived by calculating the mean and standard deviation of the overall company output in pounds and this is similar to the industrial specification Z-test ($Z_I$). However, with the industrial specification Z-test ($Z_I$) we calculated the total number of days each machine performed in 2011 and 2012 and multiplied it by the adjusted pound mix standard of each machine per day. The Z-tests are significant for the following reasons:

- The data analyzed exceeds 30 observations.
- The Z-test is used to compare the output of each machine’s performance and standard deviation to the mean of the department, company and industry specification.
- The Z-test is used to establish the process control limit.
- Create the control charts.

Simply, the Z-test is used to determine whether two population means are different when the variances are known, and the sample size is large. The Z-test is assumed to have normal distribution and nuisance parameters. The standard deviation should be known in order to accomplish accurate Z-test.

**Example of Computing the Mean and Standard Deviation**

To calculate the Z-tests, we first compute the mean and standard deviation of the data. Mean is equal to the sum of the values divided by the number of values. The equation of the observation mean is $\mu = \frac{\sum fx}{fx}$ where:

- $\mu$ = mean of the observation
- $\sum fx$ = the total sum of each value of the observation
So that the mean $\mu$ of the first 6 observation of output (pounds) from table 1 below is

$$\mu = \frac{\sum x}{6} = \frac{70042}{6} = 11673.67$$

**Computing Standard Deviation**

The observation standard deviation is how much variation or dispersion that exists from the average (mean) or expected value. This equation denotes the sample standard deviation

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N-1}}$$

Where:

$\sigma$ = the standard deviation

$x$ = each value of the observation

$\mu$ = mean of the observation

$N$ = the number of observation

From Table 1 we can calculate the standard deviation $\sigma$ and variance $\sigma^2$ of first 6 observations as follows:

$$\sqrt{\frac{(11136-11673.67)^2+(11242-11673.67)^2+(11215-11673.67)^2+(12532-11673.67)^2+(12238-11673.67)^2+(11679-11673.67)^2}{6-1}}$$

$\sigma = 590.090$

$\sigma^2 = 348,206.2$
Table 2 shows sequential manufacturing observations of nineteen individual operators with their start and end time (hours). The operators in this case worked on the first shift. Each individual operator worked with specific gauges and produced different output (pounds).

**Designing Control Chart**

As defined earlier a control chart is a statistical tool with the concept of two actions control limits A and B that lie, in general, within L1 and L2. These limits are to be set so that when the observed performance of a machine falls outside of them, even though the observation is still be within the limits L1 and L2, it is desirable to look at the manufacturing...
processes in order to discover and remove, if possible, one or more causes of variation that
need not be left to chance.26

Subsequently, the statistical theory of quality control introduces the concept of the
expected value C lying somewhere between the action limits A and B. This point C serves in
a certain sense as an aimed-at value of quality in an economically controlled state. Our
expected values will be shown shortly, in the regression parameter estimates table below.

To design the control charts:

1. Use the computed department Z-test ($Z_D$), company Z-test ($Z_C$), machine Z-test ($Z_M$)
   and industrial specification Z-test ($Z_I$) of each of the eighteen machines.

2. Compute the mean $\mu$ (tolerance) of the Z-tests for each machine.

3. Add one standard deviation $\sigma$ to the mean $\mu$ of each Z-test to obtain the upper and
   lower control limits ($\mu \pm 1\sigma$). Shewhart used larger standard deviations to design the
   control chart; however, we use 1 standard deviation $\sigma$ because it helps to detect small
   to medium shifts from the mean.

4. Add the time (hour/shift) to the Z-tests to created horizontal time series.

---

26 Ibid
### Table 3 Control Chart

<table>
<thead>
<tr>
<th>TIME (HOURS)</th>
<th>$Z_M$</th>
<th>TOLERANCE</th>
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</table>
Table 3 shows the results for creating the control chart. The tolerance is the mean and the lower and upper limits are the standard deviations (±1).
iv. Analysis

After obtaining table 3 results, we can create the control charts using the computed tolerance, and upper and lower limits of each machine. The charts enable us to visualize the performance of each machine to determine whether it’s out of control or not. The following are the control charts of eighteen machines under the department Z-test ( );

**Chart 1-CNT**

**Chart 2-CNT**

**Chart 3-RER**

**Chart 4-RUL**
The charts above indicate department Z-test ( ) on the vertical axes and 24 hours operation period on the horizontal axes. The data represent machine performances during 2011 and 2012.

Note that additional control charts of each machine with its respective company Z-test ( ), machine Z-test ( ) and industrial specification Z-test ( ) are shown in the appendix C. The department Z-test control charts will be analyze shortly.

**STD Department**

We can see that charts 15-17 above show a persistent deviation from its economic control limit on the lower half. This is what Shewhart advised to take seriously so as to eliminate it if we are to achieve an economic stable state in industrial machines. Only unit 147 of the STD is controlled around its lower limit. This means there is no indication of assignable causes around the lower half.
The Z-test of unit 036 does not deviate beyond the upper control limit. Given that most of the units of STD Department are out of control, unit 036 is controlled at its upper control limits.

We are advised out of control performance may be due to maintenance time or downtime. In addition, STD process lighter gauge materials, meaning it take more time to process material compared to units that process heavy gauge. The average gauge of STD machines within the 2011 to 2012 period is 0.069844 inches.

CNT Department

The CNT units; 022 and 023 show large deviations during the second shift. The machine performances during the third shift, were relatively within the control limit. We are informed that most of the machines continuously slowed-down during the midnight hours and may reduce quantity of output. The average gauge for the CNT machines within 2011 to 2012 period is 0.015369 inches. Our control chart shows that unit 023 produces more output than unit 022 during midnight hours.

We are advised that the schedule department and experienced worker do not work during the third shift. The scheduling used during the first shifts is rolled over to the third and second shifts, which may not necessarily meet the needs of these shifts. Scheduling is an important tool for manufacturing process where it can have a major impact on productivity. Scheduling minimizes the production time and costs, by telling a production facility when to make, with which staff, and on which equipment. Production scheduling aims to maximize the efficiency of the operation and reduce costs.
RUL Department

This department rolls metal. In metalworking, rolling is a metal forming process in which metal stock is passed through a pair of rolls. Rolling is classified according to the temperature of the metal rolled. If the temperature of the metal is above its recrystallization temperature, then the process is termed as hot rerolling. If the temperature of the metal is below its recrystallization temperature, the process is termed as cold rolling. In terms of usage, hot rolling processes more tonnage than any other manufacturing process, and cold rolling processes the most tonnage out of all cold working processes.

The RUL show a similar out of control trend to most of the STD’s machines. Units 046, 047 and 055 indicate a nonstationary trend below the lower half of the control limit. However, exhibit 2 shows that unit 046 processed more pounds of material than 047 between January-July, 2012. We predict that the efficiency level of unit 046 may be better than unit 047 in the long run.

Exhibit 2
The average gauge for the RUL machines within 2011 to 2012 period is 0.046266 inches. Unit 040 under this department shows a unique pattern which is exhibited below.

Exhibit 3

Unit 040 shows an extremely random performance among the eighteen machines analyzed. The company speculates that unit 040 was rarely in operation during the 2011 and 2012 period. Unit 040 operated under a trial basis and mostly produced scrap instead prime products. Exhibit 3 above confirms that small amounts of output were produced during second and third shifts and is stable. Stability, for the purpose of this research, is defined as when 90% of machine performance is within the control limits.

**RER Department**

RER Department is unique in the sense that, only unit 044 is selected. There were no sufficient production data for the other units of this department and therefore are eliminated. The actual average gauge of RER machines during 2011 and 2012 is 0.528265 inches. Exhibit 2 above shows that unit 044 produced the second highest amount of materials in January to July 2012.
STL Department

This department is a shearing operation that cuts large materials into narrower sizes. Potential gauges are selectively thin (0.001 to 0.215 inch) and can be machined in sheet or roll form. STL is considered a practical alternative to other methods, due to its high productivity and the versatility of materials it can manage.

The average gauge for the STL machines during 2011 and 2012 period is 0.040593 inches. The cluster analysis revealed that STL contains the highest number of machines. The performance of units 073 and 078 of this department were controlled around the lower control limits. Our investigation shows that the performance of unit 077 and 068 were relatively controlled. Units 001, 027 and 074 were totally out of control.

Machines Stability

Our control chart shows unique variations in the performance of each machine that are selected. Rarely did any of the machines perform uniformly. This confirms the thoughts of early economic quality control researchers. For industrial machines, economic quality control can be defined as a measure of predictable uniformity in performance. The assumption that industrial machines will produce the same quantity of products over time is not true.

The performances of all the machines analyzed by department and machine Z-test were out of control; however, the autocorrelation estimates predicted a stationary time series for all the machines. The variability of output per each machine is controlled under the industrial and company standard.
The following chart shows the percentage of output the machines produced above and below the limits of unit 078,001 and 027. We further explain this chart using the predictions from our regression model.

**Exhibit 5-Stable Unit**

![Control Chart Machine 078 (2011-2012)]

Unit 078 is economically stable. We define machine stability as maintaining at least 90% of output within the economic control limits. Exhibit 6 shows that 91% of the output produced by unit 78 would be stable now and in the future. In addition, the 9% of output above the control limit will remain the same in the future.

**Exhibit 6- SLT Unstable Unit**

![Control Chart Machine 001 (2011-2012)]

25% 55% 20%
Machine 001 is economically not stable. We predict 20% deviation of output will reduce in the future, and 55% would remain the same within the control limits.

**Exhibit 7-SLT stable Unit**

Unit 024 is economically stable. We predict that unit 024 will perform the same in the future. Only 4% of the output deviates below the lower half of the control limit. 95% of output is within the control limit.

**Econometrics and Regression Analysis**

The control charts show how each machine performed, given the control and tolerance limits. Here, we would consider other factors that may affect machine performance, such as the year of experience (YOE), shifts and gauge. Again Z-tests is a measure of machine performance.

The functional form of the Z-tests to be estimated is:

\[ Z\text{-tests} = f(\text{lag}(Z), \text{Dummy 1}, \text{Dummy 2}, \text{Years of Experience}, \text{Shift 1}, \text{Shift 2}, \text{Gauge}) \]
Model Specification

The procedure section derived calculation of Z-tests. Here the Z-tests are the dependent variable for the regression;

Department Z-test \((Z_D) = \frac{X_l - \mu}{\sigma_{x_l}}\)

Company Z-test \((Z_C) = \frac{X_l - \mu}{\sigma_{x_l}}\)

Industrial specification Z-test \((Z_I) = \frac{X_l - \mu}{\sigma_{x_l}}\)

Machine Z-test \((Z_M) = \frac{X_l - \mu}{\sigma_{x_l}}\)

The independent variables are:

One lag of the dependent variable \((\beta \text{lag} Z_{t-1})\)

Dummies

\(\beta D_1\) represent;

(Lag \(Z \geq 1\)) = 1
(Lag \(Z < 1\)) = 0

This means that if lag \(Z \geq 1\) and is predicted to have \(\beta D_1 > 0\), then the machine is expected to be good;

If \(Z > 1\) and is predicted to have \(\beta D_1 < 0\), then the machine is expected to be bad.

\(\beta D_2\) represent:

(Lag \(Z \leq -1\)) = 1
(Lag \(Z > -1\)) = 0

If lag \(Z \leq -1\) and is predicted to have \(\beta D_2 < 0\), then the machine is expected to be bad;

If \(Z > -1\) and is predicted to have \(\beta D_2 > 0\), then the machine is expected to be good.

Years of experience \((\beta YOE_3)\)

Shift 2 \((\beta \text{Shift } 2_4)\)
Shift 3(β_{Shift 3})

Gauge (β_{Gauge})

Note: Shift 2(β_{Shift 2}) and Shift 3 (β_{Shift 3}) are dummies as well.

The regression results show that the dummy variable βD₁ is statistically significant. Thus, given the eighteen machines’ performance, there are machines that are currently good and will continue to be good. Also, there are machines that are bad and are expected to revert back to the mean. This indicates that the firm analyzed tries to improve machine capability, even when they are not productive. We also used (Z-test ≥ 0.5) and (Z-test ≥0.25) to test the small and medium shift in the mean of the data. The results were statistically significant. The models below estimate the relationship between the Z-tests and explanatory variables. We tested the model for “goodness of fit” as well as autocorrelation using the Yule Walker and Durbin Watson test. The regression equation for predicting the dependent variable from the independent variable is;

\[ Z = \beta_0 + \beta_{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2 + \beta Shift 3 + \beta Gauge + \varepsilon \]

Where Z represent performance (pounds) for each department.

We selected the departments CNT, RER, RUL, STL and STD and predicted their gross performance using the regression equations above, although we predicted the result for the company, industrial and machine standards, to simplify, the presentation of the regression results we shall focus on the department Z-test.
Table 5-Parameter Estimate for Regression Equation:

<table>
<thead>
<tr>
<th>DEPARTMENT</th>
<th>CONSTANT</th>
<th>LAG 2</th>
<th>DUMMY</th>
<th>DUMMY 2</th>
<th>YOE</th>
<th>SHIFT 2</th>
<th>SHIFT 3</th>
<th>GAUGE</th>
<th>RSQU</th>
<th>D MEAN</th>
<th>E MEAN</th>
<th>CHI SQUARE</th>
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The horizontal headings are the independent variables, and the vertical titles are the dependent variables. We included the R-square, observed mean and expected mean. The observed mean and expected mean are used to estimate the chi square test.

**Chi Square Test**

The chi-squared test is used to test for goodness of fit of an observed distribution to a theoretical one. We used chi-square to test our regression result. The null hypothesis in this case is the relationship between observed and expected means occurred by chance. Our alternative hypothesis is that the observed and expected means are statistically significant.
Given our degree of freedom to be 6 and chi-square of 1.15, our p-value is $p<0.05$

Therefore, we can reject the null hypothesis. There is statistic significance between the dependent and expected mean.

**Interpretation**

The R-square of the CNT indicates that 26% variation of the CNT performance is predicted by the independent variable. 1% variation of the RER performance is accounted for by the independent variable. This is because the sample size of the RER was small. 67% variation of the RUL performance is predicted by the independent variable. 72% variation of the STL performance is accounted for by the independent variable. Lastly, 55% variation of the STD performance is predicted by independent variable.

Our research shows that the members of shift 1 performs better than shift 2 and shift 3. However, the control chart reveals that although shift 3 produces the least amount of output (pounds), it’s relatively controlled compare shift 2.
The analyzed firm’s observed standards are higher than what we predicted. Our expected mean, which is what one would “expect” to find if one could repeat the random variable process an infinite number of times and take the average of the values obtained, is lower compared to the firm’s standard.

This means that the firm should maintain its weighted moving average performance. This assumption is also consistent with the lag and dummy interpretation. Thus, when a machine is good in the past years, it is predicted to be even better in the future. In addition, when a machine is bad in past years, it is expected to improve. This claim is further discussed in the conclusion.

The values for the lag coefficient of all departments are negative. This indicates that changes in performance take a short time period to effect changes in the explanatory variable. The regression results show that changes in machine performance for the entire department are stationary. We present an example of stationary time series of the CNT department below.

Furthermore, we found that the STL has the largest negative t-value for YOE among the departments. For every increase in years of experience ($\beta_{YOE_3}$), there is 0.0019 decrease in the pounds produced with -22.67 t-value. This implies that different operators perform differently, based on YOE. The firm must make conscious effort to assign the appropriate experienced operator groups to machines in this department. This study grouped YOE into three categories and conducted further research, which we will discuss briefly. Gauge is statistically significant in producing higher output; the heavier the gauge, the greater
the productivity. The result of all the Z-tests calculation and tables are presented in the appendix A and B.

**Durbin Watson and Yule Walker Estimate**

Durbin–Watson Statistic is a test statistic used to detect the presence of autocorrelation (a relationship between values separated from each other by a given time lag) in the residuals (prediction errors) from a regression analysis. The Durbin-Watson Statistic ranges in value from 0 to 4. A value near 2 indicates non-autocorrelation; a value toward 0 indicates positive autocorrelation; a value toward 4 indicates negative autocorrelation.

Yule-Walker equations provide several routes to estimating the parameters of an autoregressive model by replacing the theoretical covariances with estimated values. We used this to signal the noise parameter estimates.

**Table 6- Durbin Watson and Yule Walker Test**

<table>
<thead>
<tr>
<th>DEPARTMENTAL Z-TEST</th>
<th>CONSTANT</th>
<th>DUMMY 1</th>
<th>DUMMY 2</th>
<th>YOE</th>
<th>SHIFT 2</th>
<th>SHIFT 3</th>
<th>GAUGE</th>
<th>T.RSQU</th>
<th>DURBIN WATSON</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNT Z_TEST</td>
<td>0.8808</td>
<td>1.5110</td>
<td>-0.0015</td>
<td>0.0000</td>
<td>-0.0015</td>
<td>0.0115</td>
<td>0.0172</td>
<td>0.9199</td>
<td>0.1078</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>8.35</td>
<td>13.97</td>
<td>-0.38</td>
<td>-0.41</td>
<td>-0.38</td>
<td>2.62</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RER Z_TEST</td>
<td>4.0718</td>
<td>0.4022</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0007</td>
<td>-0.0009</td>
<td>0.0000</td>
<td>0.9991</td>
<td>0.0047</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>51.09</td>
<td>14.94</td>
<td>0.00</td>
<td>-2.87</td>
<td>1.60</td>
<td>-2.01</td>
<td>-0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RUL Z_TEST</td>
<td>1.6013</td>
<td>0.1899</td>
<td>-0.6030</td>
<td>0.0000</td>
<td>0.0138</td>
<td>0.0316</td>
<td>-0.0002</td>
<td>0.3009</td>
<td>0.0565</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>242.48</td>
<td>61.13</td>
<td>-185.72</td>
<td>-0.58</td>
<td>4.26</td>
<td>6.92</td>
<td>-0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SLT Z_TEST</td>
<td>1.1415</td>
<td>0.0124</td>
<td>-0.5552</td>
<td>0.0000</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>0.0000</td>
<td>0.5715</td>
<td>0.0023</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>220.05</td>
<td>14.57</td>
<td>-649.27</td>
<td>-0.61</td>
<td>-8.08</td>
<td>-15.32</td>
<td>-0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STD Z_TEST</td>
<td>2.1613</td>
<td>0.0853</td>
<td>-0.8666</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0001</td>
<td>0.0009</td>
<td>0.9999</td>
<td>0.0359</td>
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<tr>
<td>T-VALUE</td>
<td>59.45</td>
<td>4.64</td>
<td>-43.91</td>
<td>2.05</td>
<td>-0.01</td>
<td>-0.17</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: complete copy of this table is found in the appendix
Models Fit with Autoregressive

The Durbin Watson test allowed us to describe the correlation between our variables. As you can see from the table above, the Durbin Watson Test statistics indicates serial auto correlation, since all the test statistic approach zero. We also conducted the lagged dependent variable to reduce the serial auto correlation of the residuals and hence increase the Durbin Watson statistic. The following are the Durbin Watson and Yule Walker tests of the CNT departmental Z-test.

The AUTOREG Procedure
Dependent Variable  CNT_Z_TEST

Ordinary Least Squares Estimates

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>34854.5754</td>
<td>DFE</td>
<td>47502</td>
</tr>
<tr>
<td>MSE</td>
<td>0.73375</td>
<td>Root MSE</td>
<td>0.85659</td>
</tr>
<tr>
<td>SBC</td>
<td>120184.894</td>
<td>AIC</td>
<td>120123.513</td>
</tr>
<tr>
<td>MAE</td>
<td>0.56203866</td>
<td>AICC</td>
<td>120123.516</td>
</tr>
<tr>
<td>MAPE</td>
<td>28.2606651</td>
<td>HQC</td>
<td>120142.786</td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>0.1078</td>
<td>Regress R-Square</td>
<td>0.2666</td>
</tr>
<tr>
<td>Total R-Square</td>
<td>0.2666</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parameter Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>DF</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t Value</th>
<th>Pr &gt;</th>
<th>t</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1</td>
<td>0.3985</td>
<td>0.0191</td>
<td>20.89</td>
<td>&lt;.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>1</td>
<td>1.9965</td>
<td>0.0152</td>
<td>131.10</td>
<td>&lt;.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>1</td>
<td>-0.3861</td>
<td>0.6059</td>
<td>-0.64</td>
<td>0.5240</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SHIFT_2</td>
<td>1</td>
<td>0.0319</td>
<td>0.009855</td>
<td>3.24</td>
<td>0.0012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SHIFT_3</td>
<td>1</td>
<td>0.1116</td>
<td>0.0145</td>
<td>7.69</td>
<td>&lt;.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GAUGE</td>
<td>1</td>
<td>-0.9778</td>
<td>0.5910</td>
<td>-1.65</td>
<td>0.0980</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YEARS_OF_EXPERIENCE</td>
<td>1</td>
<td>-0.000276</td>
<td>0.000370</td>
<td>-0.75</td>
<td>0.4560</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimates of Autocorrelations

| Lag | Covariance | Correlation | -1 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 |
| 0   | 0.7336     | 1.000000    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1   | 0.6494     | 0.885232    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 2   | 0.5654     | 0.770622    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 3   | 0.4814     | 0.656183    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 4   | 0.4811     | 0.655720    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 5   | 0.4808     | 0.655309    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 6   | 0.4805     | 0.654905    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 7   | 0.4798     | 0.654055    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 8   | 0.4793     | 0.653289    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 9   | 0.4787     | 0.652540    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 10  | 0.4785     | 0.652242    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

65
Preliminary MSE  0.1061

Note: the autocorrelations decrease rapidly, indicating that the change in performance is a stationary time series. Box and Narasimhan provided reasons for uncontrolled process data to be nonstationary; however, our process data is uncontrolled but stationary.

The AUTOREG Procedure

Estimates of Autoregressive Parameters

<table>
<thead>
<tr>
<th>Lag</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.986607</td>
<td>0.004465</td>
<td>-220.95</td>
</tr>
<tr>
<td>2</td>
<td>-0.000024079</td>
<td>0.006265</td>
<td>-0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.732426</td>
<td>0.006265</td>
<td>116.91</td>
</tr>
<tr>
<td>4</td>
<td>-0.721588</td>
<td>0.006775</td>
<td>-106.51</td>
</tr>
<tr>
<td>5</td>
<td>-0.000440</td>
<td>0.007213</td>
<td>-0.06</td>
</tr>
<tr>
<td>6</td>
<td>0.478826</td>
<td>0.007213</td>
<td>66.38</td>
</tr>
<tr>
<td>7</td>
<td>-0.469710</td>
<td>0.007213</td>
<td>-69.33</td>
</tr>
<tr>
<td>8</td>
<td>-0.000619</td>
<td>0.006265</td>
<td>-0.10</td>
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<tr>
<td>9</td>
<td>0.237697</td>
<td>0.005265</td>
<td>37.94</td>
</tr>
<tr>
<td>10</td>
<td>-0.230313</td>
<td>0.004465</td>
<td>-51.58</td>
</tr>
</tbody>
</table>

Yule-Walker Estimates

<table>
<thead>
<tr>
<th>SSE</th>
<th>3807.59111</th>
<th>DFE</th>
<th>47492</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.08017</td>
<td>Root MSE</td>
<td>0.28315</td>
</tr>
<tr>
<td>SBC</td>
<td>15102.4976</td>
<td>AIC</td>
<td>14953.4302</td>
</tr>
<tr>
<td>MAE</td>
<td>0.02422027</td>
<td>AICC</td>
<td>14953.4431</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.02422027</td>
<td>HQC</td>
<td>15000.236</td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>1.9427</td>
<td>Regress R-Square</td>
<td>0.0043</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total R-Square</td>
<td>0.9199</td>
</tr>
</tbody>
</table>

Parameter Estimates

| Variable               | DF | Estimate   | Standard Error | t Value | Pr > |t| |
|------------------------|----|------------|----------------|---------|------|---|
| Intercept              | 1  | 0.8808     | 0.1055         | 8.35    | <.0001|
| D1                     | 1  | 1.5110     | 0.1082         | 13.97   | <.0001|
| D2                     | 1  | -0.0365    | 0.2832         | -0.13   | 0.8974|
| SHIFT_2                | 1  | -0.001488  | 0.003903       | -0.38   | 0.7029|
| SHIFT_3                | 1  | 0.0115     | 0.004399       | 2.62    | 0.0087|
| GAUGE                  | 1  | 0.0172     | 0.1532         | 0.11    | 0.9105|
| YEARS_OF_EXPERIENCE    | 1  | -0.00035   | 0.0000856      | -0.41   | 0.6843|

We determine the noise in the data by comparing Yule Walkers’ parameter estimate with the regression parameter estimate from Table 5. The result is quiet, consistent with our original parameter estimates in terms of relationship. The dummy variables, $(\beta D_1)$ and $(\beta D_2)$, interpretation is consistent with the fact that machines are predicted to maintain,
as well as improve, performance in the future. We previously tested, using standard deviations $\sigma$ (0.25, 0.5 and 1). These deviations show that the firm’s overall machine performance during 2011 to 2012 was good. The R squares for the Yule Walker result are higher, explaining 30% to 99% of the variation in the dependent variables predicted by the independent variable for the departments. There were minute inconsistencies in signs (positive and negative) between the department Z-test and some of the explanatory variables. We conducted a further regression analysis, using SLT data to determine which group of years of experience impact productivity.

**SLT Department-Years of Experience Regression**

The cluster analysis reveals that, as operators age, their productivity level reduces. This conclusion is further supported by regression analysis. The average operator YOE is 16 years. The highest level of YOE is 45. We grouped YOE of SLT into Freshman, Junior and Senior. The Freshman is predominantly categorized to have 1-15 YOE. Junior is grouped to have 16-30 YOE, and Senior are 30-45 YOE. Our prediction is that more experience individuals are productive, however, the regression result rejects this hypothesis. The table below shows the regression result for the STL Z-test as a function of Freshmen, Junior and Senior. We also selected STL for this test, because it has the highest number of machines in this study.

Function form: $(SLT_{ztest}) = \beta_0 + \beta D_1 + \beta J_2 + \beta S_3 + \epsilon$

Where: $SLT_{ztest} = STL$ Department Z-test

$\beta X_0 = \text{intercept}$

$\beta D_1 = \text{Dummy, where } (Z\text{-test } \leq 1=1) = \text{bad performance and}$

67
(Z-test ≥1=0) = good performance

\[ \beta J_2 = \text{Junior} \]

\[ \beta S_3 = \text{Senior} \]

**Table 7: Parameter Estimate for Years of Experience**

<table>
<thead>
<tr>
<th>DEPARTMENTAL Z-TEST</th>
<th>CONSTANT</th>
<th>DUMMY</th>
<th>JUNIOR</th>
<th>SENIOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLT_Z_TEST</td>
<td>1.14699</td>
<td>-0.76680</td>
<td>0.01989</td>
<td>-0.21299</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>455.33</td>
<td>-12.18</td>
<td>5.07</td>
<td>-41.46</td>
</tr>
<tr>
<td>P-VALUE</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

The REG Procedure
Model: MODEL1
Dependent Variable: SLT_Z_TEST
Number of Observations Read 316384
Number of Observations Used 316384

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
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<td>2207.36328</td>
<td>735.78776</td>
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</tr>
<tr>
<td>Error</td>
<td>316380</td>
<td>314202</td>
<td>0.99311</td>
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</tr>
<tr>
<td>Corrected Total</td>
<td>316383</td>
<td>316409</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Root MSE 0.99655 R-Square 0.0070
Dependent Mean 1.11995 Adj R-Sq 0.0070
Coeff Var 88.98203

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|--------------------|----------------|---------|------|
| Intercept| 1  | 1.14699            | 0.00252        | 455.33  | <.0001 |
| JUNIOR   | 1  | 0.01989            | 0.00392        | 5.07    | <.0001 |
| SENIOR   | 1  | -0.21299           | 0.00514        | -41.46  | <.0001 |
| D1       | 1  | -0.76680           | 0.06295        | -12.18  | <.0001 |
The regression results indicate that Junior has greater positive impact on productivity than senior. The P-values are 0.0001, indicating that the test is statistically significant.

**Interpretation**

For every increase in Junior experience, there is 0.01989 increase in the pounds produced by the STL Department in 2011 to 2012.

For every increase in Senior experience, there is -0.21299 decrease in the pounds produced by the SLT Department in 2011 to 2012.

Given a 95% confidence level, we reject the null hypothesis that, as operator YOE increase, their performance increases. The exhibit below shows productivity by different YOE groups;

**Exhibit 4**

Exhibit 4 shows that Freshman (1-15 YOE) is the most productive, followed by Junior (16-30 YOE) and lastly Seniors (30-45 YOE). The Z-test on the vertical axe is a measure of workers’ performance. The population sample we used to reach this conclusion is 316384. Freshman is 52% of the population, Junior is 40% of this population, and Senior is
8% of the population. This study revealed that Senior performance impact productivity in the SLT Department. The reasons for such a performance could not be assigned. We urge the firm to look for the assignable causes, not only in the STL but for the other departments using our approach. YOE is one of the critical variables that needs to be looked at in the STL Department. As we noted previously, STL Department carried the highest number of machines in this study. Similar study was conducted with the CNT Department and predicted the same results.

We predict that different workers may do better on different type of machines. We recommend that the firm should further look into the variability of each machine and identify ways to reduce deviations. Although all the machines are not controlled by the department standard, they are likely to improve. We encourage that future research should consider YOE as one of the key variables to analyze these machines. Periodic shuffling of operators should be encouraged, since many experienced individuals may be unproductive operating a particular machine. Future research should measure the rate of stability of the all machinery and further analyze these machines using variables, such as downtime, machine schedule and run times, to predict the actual causes of deviation and eliminate them.
v. Conclusion

I have developed the “New Synergetic Trilogy Control” to find the most economical design for Shewhart-type control charts and have applied it to a large North American manufacturing firm. The New Synergetic Trilogy Control accounts for extensive application of applied economics, statistic and engineering processes. Our four simple steps to create the control chart is a contribution to the improvement of pure statistical design. The method is less costly and is marketable under the assumptions and properties of economic quality control.

This statistical design identified small to medium shifts in machines, and makes predictions on how a machine’s performance would vary in the future. It simultaneously identifies out-of-control performance and stability in any machine. Our control chart is robust, it breaks down each shift during which an operator worked with a machine, and even identifies a machine that was not in operation during 2011 and 2012 production period. Our model found that a data process can be uncontrolled and be stationary which is contrary to Box and Narasimhan’s claims.

We introduced the concept of developing company, machine, department and industrial standards to re-enforce Shewhart’s idea of sub-grouping to detect assignable causes of variation. Our regression shows that, regardless of the standards, a good machine would maintain its performance, and a bad machine would improve. In addition, we confirmed that the firm’s current moving averages are good.

This research is important because it the first of its kind conducted on the machines of this Northeast American firm. Industrial machine performance is an important issue facing
US companies, but it was not until the 1980’s that the United States imported the concept from Japan and was not taught in universities until 1993. In this global business environment, manufacturing organization cannot compete successfully without some sort of quality product. This was evident in the 1950’s where quality control improved the performance of manufacturing industries in Japan.

We reviewed the literature and philosophies of the inception of economic quality control and have remarked on the concepts and application. Nevertheless, we should appreciate the enormous works of early statisticians, although there is still evident need for industrial machine process optimality and fundamental improvement. This can only be achieved through statistical evidence.
Bibliography


Appendix A- Complete Regression Parameter Estimates

Table 1 shows the parameter estimates of the regression equation.
\[ Z = \beta_0 + \text{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta \text{Shift} 2 + \beta \text{Shift} 3 + \beta \text{Guage} \]

The columns represent the dependent and independent variables. There are five departments under four statistical models. This is found on the first column. The rows show the actual estimates of the departments with its corresponding t-test values. The last four columns represent the estimates for R-square, dependent mean, expected mean and chi-square, respectively.
### Yule Walker and Durbin Watson Table

<table>
<thead>
<tr>
<th>DEPARTMENTAL Z-TEST</th>
<th>CONSTANT DUMMY 1 DUMMY 2 YOE SHIFT 2 SHIFT 3 GAUGE T.RSQU DURBIN WATSON</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNT_Z_TEST</td>
<td>0.8808 1.5110 -0.0015 0.0000 -0.0015 0.0115 0.0172 0.9199 0.1078</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>8.35 13.97 -0.38 -0.41 -0.38 2.62 0.11</td>
</tr>
<tr>
<td>RER_Z_TEST</td>
<td>4.0718 0.4027 0.0000 0.0000 0.0007 -0.0009 0.0000 0.9991 0.0047</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>51.09 14.94 0.00 -2.87 1.60 -2.01 -0.04</td>
</tr>
<tr>
<td>RUL_Z_TEST</td>
<td>1.6013 0.1899 -0.6030 0.0000 0.0138 0.0316 -0.0002 0.3009 0.0565</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>242.48 61.13 -185.72 -0.58 4.26 6.92 -0.33</td>
</tr>
<tr>
<td>SLT_Z_TEST</td>
<td>1.1415 0.0124 -0.5552 0.0000 -0.0001 -0.0001 0.0000 0.5715 0.0023</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>220.05 14.57 -649.27 -0.80 -8.08 -15.32 -0.02</td>
</tr>
<tr>
<td>STD_Z_TEST</td>
<td>2.1613 0.0853 -0.8666 0.0000 0.0000 -0.0001 0.0000 0.9999 0.0359</td>
</tr>
<tr>
<td>T-VALUE</td>
<td>59.45 4.64 -43.91 2.05 -0.01 -0.17 0.45</td>
</tr>
</tbody>
</table>

| COMPANY Z-TEST      | CNT_Z_TEST 7.8998 -1.0168 0.0000 -0.0001 -0.0001 0.0006 0.9998 0.0014 |
| T-VALUE             | 61.96 14521.80 -1921.90 4.29 -6.81 -9.50 1.60 |
| RER_Z_TEST          | 2.4115 0.0085 -135.1825 0.0000 0.0000 -0.0005 0.0000 0.9995 0.0046 |
| T-VALUE             | 56.06 0.58 -849.47 -2.97 1.57 -2.01 -0.04 |
| RUL_Z_TEST          | 0.1247 1.9815 -2.8980 0.0001 0.0015 0.0012 -0.0572 0.9326 0.1638 |
| T-VALUE             | 2.91 41.57 -15.79 1.08 0.55 0.46 -1.99 |
| SLT_Z_TEST          | 0.4074 1.3670 -0.4890 0.0000 -0.0002 -0.0001 -0.0008 0.9978 0.0148 |
| T-VALUE             | 38.54 88.49 -12.06 -2.42 -0.55 -0.51 -0.52 |
| STD_Z_TEST          | 0.5130 1.3670 -0.4890 0.0000 -0.0001 -0.0001 0.0000 0.9806 0.0744 |
| T-VALUE             | 15.73 4.64 -43.91 2.05 -0.01 -0.17 0.45 |

| MACHINE Z-TEST      | CNT_Z_TEST 0.8298 1.6817 -0.0480 0.0000 0.0011 0.0152 -0.0442 0.9469 0.1187 |
| T-VALUE             | 8.71 17.17 -0.19 -0.36 0.28 3.32 -0.25 |
| RER_Z_TEST          | 3.8882 0.5161 0.0000 0.0000 0.0516 0.1128 0.0000 0.9991 0.0047 |
| T-VALUE             | 53.89 21.46 -0.00 1.19 2.18 1.38 0.00 |
| RUL_Z_TEST          | 1.8835 0.0139 -0.4856 0.0000 -0.0001 -0.0002 0.0003 0.0887 0.0043 |
| T-VALUE             | 2.91 41.57 -15.79 1.08 0.55 0.46 -1.99 |
| SLT_Z_TEST          | 0.4074 1.3670 -0.4890 0.0000 -0.0002 -0.0001 -0.0008 0.9978 0.0148 |
| T-VALUE             | 38.54 88.49 -12.06 -2.42 -0.55 -0.51 -0.52 |
| STD_Z_TEST          | 0.5130 1.3670 -0.4890 0.0000 -0.0001 -0.0001 0.0000 0.9806 0.0744 |
| T-VALUE             | 15.73 4.64 -43.91 2.05 -0.01 -0.17 0.45 |

| INDUSTRIAL SPEC     | CNT_Z_TEST 6.22E+12 2.12E+13 -5.16E+14 -4.83E+08 1.16E+11 6.86E+11 -9.76E+11 0.922 0.1253 |
| T-VALUE             | 3.70E+00 1.25E+01 -1.38E+02 -3.70E+01 1.04E+00 5.71E+00 -4.20E+01 |
| RER_Z_TEST          | 1.51E+12 8.10E+09 -8.40E+13 -1.10E+07 1.13E+09 -3.91E+09 -3.43E+04 0.9995 0.0017 |
| T-VALUE             | 2.92E+01 8.90E+01 -8.27E+03 -3.34E+00 2.79E+00 -9.63E+00 -1.00E+02 |
| RUL_Z_TEST          | -1.30E+12 5.49E+12 -2.97E+13 -5.40E+08 -2.90E+10 -3.04E+10 -6.10E+10 0.8762 0.1931 |
| T-VALUE             | -2.71E+00 1.06E+01 -1.63E+01 -7.60E+01 -5.10E+01 -5.40E+01 -2.00E+01 |
| SLT_Z_TEST          | 3.02E+12 7.96E+11 -1.25E+12 -1.40E+07 -2.97E+09 -5.99E+09 -3.17E+09 0.9964 0.0006 |
| T-VALUE             | 1.17E+01 3.67E+00 -5.73E+00 -3.50E+01 -7.50E+01 -1.53E+00 -3.30E+01 |
| STD_Z_TEST          | 1.24E+12 4.18E+12 -4.15E+13 -2.52E+07 -5.32E+09 -9.68E+09 5.62E+10 0.9729 0.2256 |
| T-VALUE             | 3.75E+00 1.26E+01 -8.74E+01 1.00E-01 3.90E-01 -7.00E-01 1.02E+00 |

The table below represents the Yule Walker estimate and Durbin Watson test. These tests are conducted to test for auto correlation and “goodness of fit” of the model. The
columns show dependent and independent variables with R-squares and the Durban Watson test. The rows are the actual estimate of the five dependent variables, categorized into four statistical models.
Appendix B- Computations for Expected Mean

Appendix B shows the detailed calculations of expected mean. The expected mean is computed for the five departments grouped by department Z-test, Company Z-test, Industrial Specification Z-test and Machine Z-test. The average pound for the entire observation is 10851.17307lb.

\[ E(Z_D) \] represent expected value of department Z-test

\[ E(Z_C) \] represent expected value of company Z-test

\[ E(Z_I) \] represent expected value of the industrial specification Z-test

\[ E(Z_M) \] represent expected value of machine Z-test

\[ CNT \]

\[ E(Z_D) = \beta_0 + \beta_{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 4 + \beta Shift 3 + \beta Guage + \epsilon \]

\[ \text{Averages} \]
\[ \text{Pounds}= 10851.17307lb -5ton \]
\[ \text{Dependent Mean}=2.24992 \]
\[ \beta_0 = 0.3985 \]
\[ \beta_{lag}Z_{t-1}=0 \]
\[ \beta D_1= 0 \]
\[ \beta D_2= 1 \]
\[ \beta YOE_3=16 \]
\[ \beta Shift 4=0 \]
\[ \beta Shift 3=1 \]
\[ \beta Guage = 0.015369 \]

\[ E(Z_D)= 0.3985+ 0(-0.9866) +0(1.9965)+1(-0.3861)+16(-0.0003) +0(0.0319)+1(0.1116)+0.015369(-0.9778) \]

\[ E(Z_D)= 0.3985+0+0-0.3861-0.0048+0+0.1116-0.0150 \]

\[ E(Z_D)= 0.1042 \]
RER

\[ E(Z_D) = \beta_0 + \beta_{\text{lag}Z_{t-1}} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta \text{Shift 2}_4 + \beta \text{Shift 3}_5 + \beta \text{Guage} + \epsilon \]

\textbf{Averages}

\textit{Pounds= 10851.17307lb -5ton}
Dependent Mean= 4.4102
\[ \beta_0 = 0.2272 \]
\[ \beta_{\text{lag}Z_{t-1}} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta YOE_3 = 16 \]
\[ \beta \text{Shift 1}_4 = 0 \]
\[ \beta \text{Shift 2}_5 = 1 \]
\[ \beta \text{Guage} = 0.528265 \]

\[ E(Z_D) = 0.2272 + 0(-0.8944) + 0(4.2086) + 1(0.0000) + 16(0.0018) + 1(-0.0087) + 1(-0.1737) + 0.528265(0.0001) \]

\[ E(Z_D) = 0.2272 + 0 + 0 + 0.0288 + 0 - 0.1737 + 0.000052827 \]

\[ E(Z_D) = 0.0824 \]

\textit{RUL}

\[ E(Z_D) = \beta_0 + \beta_{\text{lag}Z_{t-1}} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta \text{Shift 2}_4 + \beta \text{Shift 3}_5 + \beta \text{Guage} + \epsilon \]

\textbf{Averages}

\textit{Pounds= 10851.17307lb -5ton}
Dependent Mean= 1.7850
\[ \beta_0 = -0.1008 \]
\[ \beta_{\text{lag}Z_{t-1}} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta YOE_3 = 16 \]
\[ \beta \text{Shift 1}_4 = 0 \]
\[ \beta \text{Shift 2}_5 = 1 \]
\[ \beta \text{Guage} = 0.046266 \]

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\[ E(Z_D) = -0.1008 + 0(-0.2620) + 0(2.0254) + 1(-0.7716) + 16(0.0012) + 1(0.0789) + 0.046266 (3.4845) \]
\[ E(Z_D) = -0.1008 + 0 + 0 - 0.7716 + 0 + 0.0789 + 0.1612 \]
\[ E(Z_D) = -0.6131 \]

**SLT**

\[ E(Z_D) = \beta_0 + \beta_{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta Guage + \varepsilon \]

**Averages**

**Pounds = 10851.17307lb - 5ton**
Dependent Mean = 1.1200

\[ \beta_0 = 0.3529 \]
\[ \beta_{lag}Z_{t-1} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta YOE_3 = 16 \]
\[ \beta Shift 2_4 = 0 \]
\[ \beta Shift 3_5 = 1 \]
\[ \beta Guage = 0.040593 \]

\[ E(Z_D) = 0.3529 + 0(-0.5813) + 0(1.6969) + 1(-1.1021) + 16(-0.0019) + 0(-0.0223) + 1(0.0212) + 0.040593 (-0.1168) \]
\[ E(Z_D) = 0.3529 + 0 + 0 - 1.1021 - 0.0304 + 0 + 0.0212 - 0.005 \]
\[ E(Z_D) = -0.7634 \]

**STD**

\[ E(Z_D) = \beta_0 + \beta_{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta Guage + \varepsilon \]

**Averages**

**Pounds = 10851.17307lb - 5ton**
Dependent Mean = 2.1921

\[ \beta_0 = 0.1850 \]
\[ \beta_{lag}Z_{t-1} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta YOE_3 = 16 \]
\[ \beta Shift 1_4 = 0 \]
\[ \beta Shift 2_5 = 1 \]
\[ \beta Guage = 0.069844 \]

\[ E(Z_D) = 0.1850 + 0(-0.3227) + 0(2.0476) + 1(-1.2329) + 16(0.0051) + 0(0.0837) + 1(0.0810) + 0.069844(1.5712) \]
\[ E(Z_D) = 0.1850 + 0 - 1.2329 + 0.0816 + 0 + 0.0810 + 0.1097 \]
\[ E(Z_D) = -0.7756 \]

**Company**

**CNT**

\[ E(Z_C) = \beta_0 + \beta_{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 1_4 + \beta Shift 2_5 + \beta Guage + \epsilon \]

**Averages**

**Pounds = 10851.17307lb -5ton**

Dependent Mean = 0.2835

\[ \beta_0 = 0.2775 \]
\[ \beta_{lag}Z_{t-1} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta YOE_3 = 16 \]
\[ \beta Shift 1_4 = 0 \]
\[ \beta Shift 2_5 = 1 \]
\[ \beta Guage = 0.015369 \]

\[ E(Z_C) = 0.2775 + 0(-0.8294) + 0(8.2372) + 1(-1.3229) + 16(0.000049) + 0(0.0001) + 1(0.0049) + 0.015369(0.3807) \]
\[ E(Z_C) = 0.2775 + 0 - 1.3229 + 0.000784 + 0 + 0.0049 + 0.005851 \]
\[ E(Z_C) = -1.034 \]
RER

\[ E(Z_c) = \beta_0 + \beta_{\text{lag}Z_{t-1}} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta \text{Shift}_4 + \beta \text{Shift}_5 + \beta \text{Guage} + \epsilon \]

Averages

\begin{align*}
\text{Pounds} &= 10851.17307\text{lb -5ton} \\
\text{Dependent Mean} &= 2.3877 \\
\beta_0 &= 0.8414 \\
\beta_{\text{lag}Z_{t-1}} &= 0 \\
\beta D_1 &= 0 \\
\beta D_2 &= 1 \\
\beta YOE_3 &= 16 \\
\beta \text{Shift}_4 &= 0 \\
\beta \text{Shift}_5 &= 1 \\
\beta \text{Guage} &= 0.528265 \\
\end{align*}

\[ E(Z_c) = 0.8414 \times 0 + (-0.8954) + 0(1.5634) + 1(-136.0588) + 0(0.0010) + (0(-0.0033)) + 1(-0.0918) + \\
0.528265(0.0000387) \]

\[ E(Z_c) = 0.8414 + 0 + 0 -136.0588 + 0.016 + 0 -0.0033 -0.0918 + 0.0000204 \]

\[ E(Z_c) = -135.30 \]

RUL

\[ E(Z_c) = \beta_0 + \beta_{\text{lag}Z_{t-1}} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta \text{Shift}_4 + \beta \text{Shift}_5 + \beta \text{Guage} + \epsilon \]

Averages

\begin{align*}
\text{Pounds} &= 10851.17307\text{lb -5ton} \\
\text{Dependent Mean} &= 1.7301 \\
\beta_0 &= -0.0498 \\
\beta_{\text{lag}Z_{t-1}} &= 0 \\
\beta D_1 &= 0 \\
\beta D_2 &= 1 \\
\beta YOE_3 &= 16 \\
\beta \text{Shift}_4 &= 0 \\
\beta \text{Shift}_5 &= 1 \\
\beta \text{Guage} &= 0.046266 \\
\end{align*}

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\[ E(Z_c) = -0.0498 + 0(-0.3907) + 0(1.9547) + 1(-8.3486) + 16(0.0022) + 0(0.0825) + 1(0.0331) + 0.046266 (3.2472) \]
\[ E(Z_c) = -0.0498 + 0 + 0 - 8.3486 + 0.0352 + 0.0331 + 0.1502 \]
\[ E(Z_c) = -8.1799 \]

**SLT**

\[ E(Z_c) = \beta_0 + \beta_{lag} Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta Guage + \varepsilon \]

\begin{align*}
\text{Averages} \\
Pounds &= 10851.17307\text{lb} - 5\text{ton} \\
\text{Dependent Mean} &= 1.0053 \\
\beta_0 &= 0.3817 \\
\beta_{lag} Z_{t-1} &= 0 \\
\beta D_1 &= 0 \\
\beta D_2 &= 1 \\
\beta YOE_3 &= 16 \\
\beta Shift 1_4 &= 0 \\
\beta Shift 2_5 &= 1 \\
\beta Guage &= 0.040593
\end{align*}

\[ E(Z_c) = 0.3817 + 0(-0.9526) + 0(1.5633) + 1(-9.8602) + 16(-0.0020) + 0(-0.0018) + 1(0.0276) + 0.040593(-0.4809) \]
\[ E(Z_c) = 0.3817 + 0 + 0 - 9.8602 + 0.032 + 0 + 0.0276 - 0.01952 \]
\[ E(Z_c) = -9.438 \]

**STD**

\[ E(Z_c) = \beta_0 + \beta_{lag} Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta Guage + \varepsilon \]

\begin{align*}
\text{Averages} \\
Pounds &= 10851.17307\text{lb} - 5\text{ton} \\
\text{Dependent Mean} &= 1.7849 \\
\beta_0 &= 0.2972
\end{align*}
\[ \beta_{\text{lag}Z_{t-1}} = 0 \]
\[ \beta_D_1 = 0 \]
\[ \beta_D_2 = 1 \]
\[ \beta_{\text{YOE}_3} = 16 \]
\[ \beta_{\text{Shift }1_4} = 0 \]
\[ \beta_{\text{Shift }2_5} = 1 \]

\[ \beta_{\text{Guage}} = 0.069844 \]

\[ \text{E}(Z_C) = 0.2972 + 0(-0.8623) + 0(1.5841) + 1(-12.3329) + 16(0.0041) + 0(0.0641) + 1(0.0598) + 0.069844 (1.1617) \]

\[ \text{E}(Z_C) = 0.2972 + 0 + 0 - 12.3329 + 0.0656 + 0 + 0.0598 + 0.08114 \]

\[ \text{E}(Z_C) = -11.829 \]

**Machine**

**CNT**

\[ \text{E}(Z_M) = Z_D = \beta_0 + \beta_{\text{lag}Z_{t-1}} + \beta_D_1 + \beta_D_2 + \beta_{\text{YOE}_3} + \beta_{\text{Shift }2_4} + \beta_{\text{Shift }3_5} + \beta_{\text{Guage}} + \epsilon \]

**Averages**

**Pounds = 10851.17307lb -5ton**

**Dependent Mean = 2.3461**

\[ \beta_0 = 0.1457 \]
\[ \beta_{\text{lag}Z_{t-1}} = 0 \]
\[ \beta_D_1 = 0 \]
\[ \beta_D_2 = 1 \]
\[ \beta_{\text{YOE}_3} = 16 \]
\[ \beta_{\text{Shift }1_4} = 0 \]
\[ \beta_{\text{Shift }2_5} = 1 \]

\[ \beta_{\text{Guage}} = 0.015369 \]

\[ \text{E}(Z_M) = 0.1457 + 0(-0.8494) + 0(2.0086) + 1(-0.5128) + 16(-0.0024) + 0(-0.1988) + 1(-0.1014) + 0.015369 (28.7344) \]
\[ E(Z_M) = 0.1457 + 0 + 0 -0.5128 + 0.0384 + 0 - 0.1014 + 0.44162 \]

\[ E(Z_M) = 0.01152 \]

**RER**

\[ E(Z_M) = Z_D) = \beta_0 + \beta lag Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta Guage + \varepsilon \]

**Averages**

**Pounds= 10851.17307lb -5ton**

Dependent Mean= 4.4102

\[ \beta_0= 0.1846 \]
\[ \beta lag Z_{t-1}=0 \]
\[ \beta D_1= 0 \]
\[ \beta D_2= 1 \]
\[ \beta YOE_3=16 \]
\[ \beta Shift 1_4=0 \]
\[ \beta Shift 2_5=1 \]

\[ \beta Guage = 0.528265 \]

\[ E(Z_M) = 0.1846 + 0 (-0.9626) + 0(3.1993)+ 1(0.0000) + 16(0.0006) + 0(0.8734) + 1(2.2163) + 0.528265 \]

\[ E(Z_M) = 0.1846 + 0 + 0 + 0 + 0.0096 + 0 + 2.2163 + 0.000264 \]

\[ E(Z_M) = 2.410 \]

**RUL**

\[ E(Z_M) = Z_D) = \beta_0 + \beta lag Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta Guage + \varepsilon \]

**Averages**

**Pounds= 10851.17307lb -5ton**

Dependent Mean= 1.9214

\[ \beta_0= 0.0395 \]
\[ \beta lag Z_{t-1}=0 \]
\[ \beta D_1= 0 \]
\[ \beta D_2 = 1 \]
\[ \beta YOE_3 = 16 \]
\[ \beta \text{Shift 1}_4 = 0 \]
\[ \beta \text{Shift 2}_5 = 1 \]
\[ \beta \text{Guage} = 0.046266 \]

\[ E(Z_M) = 0.0395 + 0(-0.5091) + 0(2.2314) + 1(-1.2640) + 16(0.0012) + 0(0.0382) + 1(0.0026) + 0.046266(0.6755) \]

\[ E(Z_M) = 0.0395 + 0 + 0 + 0 - 1.2640 + 0.0192 + 0.0026 + 0.03125 \]

\[ E(Z_M) = -1.17145 \]

### SLT

\[ E(Z_M) = Z_D = \beta_0 + \beta \text{lag} Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta \text{Shift 2}_4 + \beta \text{Shift 3}_5 + \beta \text{Guage} + \epsilon \]

**Averages**

**Pounds = 10851.17307lb - 5ton**

Dependent Mean = 1.7849

\[ \beta_0 = 0.3486 \]
\[ \beta \text{lag} Z_{t-1} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta YOE_3 = 16 \]
\[ \beta \text{Shift 1}_4 = 0 \]
\[ \beta \text{Shift 2}_5 = 1 \]
\[ \beta \text{Guage} = 0.040593 \]

\[ E(Z_M) = 0.3486 + 0(-0.6818) + 0(1.8965) + 1(-1.2127) + 16(-0.0011) + 0(0.0738) + 1(0.0079) + 0.040593(-0.6954) \]

\[ E(Z_M) = 0.3486 + 0 + 0 - 1.2127 - 0.0176 + 0 + 0.0079 - 0.0282 \]

\[ E(Z_M) = -0.902 \]
STD

\[ E(Z_M) = Z_D = \beta_0 + \beta_{\text{lag} Z_{t-1}} + \beta D_1 + \beta D_2 + \beta_{\text{YOE}_3} + \beta_{\text{Shift } 2_4} + \beta_{\text{Shift } 3_5} + \beta_{\text{Guage}} + \epsilon \]

Averages

**Pounds**: 10851.17307lb - 5ton
Dependent Mean: 2.5501

\[ \beta_0 = 0.2821 \]
\[ \beta_{\text{lag} Z_{t-1}} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta_{\text{YOE}_3} = 16 \]
\[ \beta_{\text{Shift } 1_4} = 0 \]
\[ \beta_{\text{Shift } 2_5} = 1 \]
\[ \beta_{\text{Guage}} = 0.069844 \]

\[ E(Z_M) = 0.2821 + 0(-0.4102) + 0(2.4676) + 1(-2.0091) + 16(-0.0026) + 0(-0.0438) + 1(-0.0387) + 0.069844(1.8389) \]

\[ E(Z_M) = 0.2821 + 0 + 0 - 2.0091 - 0.0416 + 0 - 0.0387 + 0.1284 \]

\[ E(Z_M) = -1.6789 \]

**Industrial Specification**

CNT

\[ E(Z_1) = \beta_0 + \beta_{\text{lag} Z_{t-1}} + \beta D_1 + \beta D_2 + \beta_{\text{YOE}_3} + \beta_{\text{Shift } 2_4} + \beta_{\text{Shift } 3_5} + \beta_{\text{Guage}} + \epsilon \]

Averages

**Pounds**: 10851.17307lb - 5ton
Dependent Mean: 2.64E+13

\[ \beta_0 = 1.74E+12 \]
\[ \beta_{\text{lag} Z_{t-1}} = 0 \]
\[ \beta D_1 = 0 \]
\[ \beta D_2 = 1 \]
\[ \beta_{\text{YOE}_3} = 16 \]
\[ \beta_{\text{Shift } 1_4} = 0 \]
\[ \beta_{\text{Shift } 2_5} = 1 \]
$\beta_{Guage} = 0.015369$

$$E(Z_t) = 1736600 + 0(-0.915604) + 0(28166000) + 1(-572100000)+16(12001) + 0(2226200) + 1(3119900) + 0.015369(-22940000)$$

$$E(Z_t) = 1736600 + 0 + 0 -572100000 + 192016 + 0 + 3119900 -352564.86$$

$$E(Z_t) = 56740404900000$$

**RER**

$$E(Z_t) = \beta_0 + \beta_{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta_{Guage} + \epsilon$$

**Averages**

*Pounds= 10851.17307lb -5ton*  
Dependent Mean= 2.64E+13

$$\beta_0 = 1.74E+12$$  
$$\beta_{lag}Z_{t-1} = 0$$  
$$\beta D_1 = 0$$  
$$\beta D_2 = 1$$  
$$\beta YOE_3 = 16$$  
$$\beta Shift 1_4 = 0$$  
$$\beta Shift 2_5 = 1$$  
$$\beta_{Guage} = 0.528265$$

$$E(Z_t) = 214450 + 0(-0.926446) + 0(14717000) + 1(-840500000) + 16(587991599) + 0(-27833) + 1(-583600) + 0.528265(36996305)$$

$$E(Z_t) = 214450 + 0 + 0 -840500000 + 9407865584 + 0 -583600 + 19543853.06$$

$$E(Z_t) = 8586540287$$

**RUL**

$$E(Z_t) = \beta_0 + \beta_{lag}Z_{t-1} + \beta D_1 + \beta D_2 + \beta YOE_3 + \beta Shift 2_4 + \beta Shift 3_5 + \beta_{Guage} + \epsilon$$

**Averages**

*Pounds= 10851.17307lb -5ton*
Dependent Mean= 3.42E+12
\( \beta_0 = 1.74E+12 \)
\( \beta_{lagZ_{t-1}} = 0 \)
\( \beta D_1 = 0 \)
\( \beta D_2 = 1 \)
\( \beta_{YOE_3} = 16 \)
\( \beta_{Shift 1_4} = 0 \)
\( \beta_{Shift 2_5} = 1 \)
\( \beta_{Guage} = 0.046266 \)

\[ E(Z_t) = 332070 + 0(-0.380353) + 0(439240) + 1(-10550000) + 16(-1236) + 0(-136400) + 1(-121800) + 0.046266(-5719000) \]

\[ E(Z_t) = 332070 + 0 + 0 -1055000 + 19776 + 0 -121800 -264595.25 \]

\[ E(Z_t) = -1089549 \]

SLT

\[ E(Z_t) = \beta_0 + \beta_{lagZ_{t-1}} + \beta D_1 + \beta D_2 + \beta_{YOE_3} + \beta_{Shift 2_4} + \beta_{Shift 3_5} + \beta_{Guage} + \epsilon \]

Averages

Pounds= 10851.17307lb -5ton
Dependent Mean= 4.11E+12

\( \beta_0 = 3.57E+11 \)
\( \beta_{lagZ_{t-1}} = 0 \)
\( \beta D_1 = 0 \)
\( \beta D_2 = 1 \)
\( \beta_{YOE_3} = 16 \)
\( \beta_{Shift 1_4} = 0 \)
\( \beta_{Shift 2_5} = 1 \)
\( \beta_{Guage} = 0.040593 \)

\[ E(Z_t) = 35696 + 0(-0.779716) + 0(409570) + 1(-4589000) + 16(-1321) + 0(-12690) + 1(72045) + 0.040593(-267200) \]
\[ E(Z_t) = 35696 + 0 + 0 \cdot 4589000 - 21136 + 0 + 72045 - 10846.4496 \]

\[ E(Z_t) = -4513240 \]

**STD**

\[ E(Z_t) = \beta_0 + \beta_{lag}Z_{t-1} + \beta_{D1} + \beta_{D2} + \beta_{YOE3} + \beta_{Shift 24} + \beta_{Shift 35} + \beta_{Guage} + \epsilon \]

- **Averages**
  - **Pounds** = 10851.17307lb - 5ton
  - Dependent Mean = 5.28E+12
    - \( \beta_0 = 1.60E+12 \)
    - \( \beta_{lag}Z_{t-1} = 0 \)
    - \( \beta_{D1} = 0 \)
    - \( \beta_{D2} = 1 \)
    - \( \beta_{YOE3} = 16 \)
    - \( \beta_{Shift 14} = 0 \)
    - \( \beta_{Shift 25} = 1 \)
    - \( \beta_{Guage} = 0.069844 \)

\[ E(Z_t) = 159970 + 0 \cdot (-0.379638) + 0(590900) + 1(-5785000) + 1(13454) + 0(14055) + 1(-6585) + 0.069844(-2982000) \]

\[ E(Z_t) = -5624626 \]
Appendix C- Additional Charts

Appendix C shows 54 control charts for all machines. All eighteen department control charts are presented early in chapter IV.
RER

Control Chart Machine 044 (2011-2012)

Control Chart Machine 044 (2011-2012)

Control Chart Machine 023 (2011-2012)

Control Chart Machine 023 (2011-2012)

Control Chart Machine 044 (2011-2012)

Control Chart Machine 044 (2011-2012)
STD

Control Chart Machine 036 (2011-2012)

Control Chart Machine 036 (2011-2012)

Control Chart Machine 036 (2011-2012)

Control Chart Machine 133 (2011-2012)
STL